Tampere University

HARRI NIKULA

Subsidized Prices vs. Subsidized Quantities

Choosing Simple Instruments in Challenging Policy Regimes

Tampere University Dissertations 141

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ACADEMIC DISSERTATION
To be presented, with the permission of
the Faculty Council of the Faculty of Management and Business
of Tampere University,
for public discussion in the Paavo Koli -Auditorium
of the Pinni A Building, Kanslerinrinne 1, Tampere,
on 25 October 2019, at 12 o’clock.

ACADEMIC DISSERTATION
Tampere University, Faculty of Management and Business
Finland

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The originality of this thesis has been checked using the Turnitin OriginalityCheck service.

Cover design: Roihu Inc.

ISBN 978-952-03-1277-0 (print)
ISBN 978-952-03-1278-7 (pdf)
ISSN 2489-9860 (print)
ISSN 2490-0028 (pdf)
http://urn.fi/URN:ISBN:978-952-03-1278-7

PunaMusta Oy – Yliopistopaino
Tampere 2019

ACKNOWLEDGMENTS

I thank Pauli Murto and Matti Liski for their good advice and ideas for an aspiring doctoral student. I thank Pauli Lappi and Timo Tohmo for their comments on early versions of this work. I also thank Hannu Laurila and Sami Remes for helping with the presentational details of scientific writing. I thank my pre-examiners, Jari Kaivo-oja and Sakari Uimonen, for their helpful comments and suggestions.

Several institutions have provided financial support for this work. I thank Nordic Energy Research, Finnish Doctoral Research in Economics, Tampere University, and the Yrjö Jahnsson Foundation for providing an opportunity to conduct long-term academic work.

I have been working for the last fifteen years with many economists at Tampere University. You have a surprising amount in common, as you are all talented and good people. Thank you for your patience and for being role models who have guided me toward this goal.

Finally, my deepest thanks go to my family: to my parents, Raija and Erkki; to my wife, Marika; and to my children, Sauli, Onni, and Helka.

ABSTRACT

This thesis studies the goals and means for reducing emissions. It consists of theoretical models that apply and extend the classical model that Martin L. Weitzman introduced in 1974. The Weitzman model restricts the means of regulation to price and quantity policies, which require setting either a price or a quota in the market for emissions production. Weitzman finds that the instruments are not equivalent, as asymmetric information induces societal costs that differ between price and quantity policies. In the thesis, the Weitzman model is extended by incorporating discrete choices, subsidization, and cost inefficiencies. The augmented models demonstrate that the new features change the classical instrument choice formula in significant ways.

A discrete choice is a major project that produces considerable emission reductions with a considerable monetary outflow; an investment in green technology is an example of a discrete choice. A regulated firm is subsidized if it does not have to pay for every emission unit it produces. Taken together, the thesis discusses how subsidization manipulates emissions payments and discrete choices such that aggregate emissions reduction becomes inefficiently organized. Importantly, the thesis studies tradable permits (quantity policy) and environmental taxes (price policy) that both contain emission payments, so the instruments are vulnerable to subsidization and inefficiencies. The thesis traces instances of subsidization and subsequent cost inefficiency in environmental policy and discusses two general sources of inefficiencies in more detail: technical factors and political factors.

Political inefficiency is the result of conflict between the regulator and various policy stakeholders. The regulator promotes the efficient use of natural resources, whereas the stakeholders often have other (usually narrower) goals—incumbent polluting firms seek tax exemptions while environmentalists aim for generous support for green technologies.

Technical inefficiency is a more subtle concept and refers to the conflict that arises

between the complexity of the regulated industries and the simplicity of the regulatory instruments. In the thesis, an important source of complexity is that technological externalities create agglomeration economies, so, as more companies invest in green technology, additional investments become less expensive. Whenever technological externalities interact with asymmetric information, the thesis shows that an efficient solution involves subsidizing firms, but the policy requires very complicated subsidization designs.

The extensions of the original Weitzman model produce novel insights into the rules for the choice of instrument between price policy and quantity policy. The thesis isolates three general effects that influence the classical instrument choice: the base effect, cost effect, and volume effect. Combining these effects, the thesis shows that the overall effect on instrument choice depends on the policy- and industry-specific factors.

The base effect originates from changes in the regulated industries' cost structures in the absence of inefficiencies. The thesis discusses two particular sources of the base effect: programs encouraging voluntary emissions reductions and the presence of positive technological externalities. In both cases, the aggregate abatement costs fall. The thesis argues that the regulator now pays more attention to the damages caused by pollution, favoring the use of pollution quotas. Therefore, the base effect is unilaterally biased toward quantity regulation.

The cost effect is primarily the result of inefficient subsidization of polluting firms. The thesis shows that the shape of the cost effect depends on the source of the inefficiency. However, whether inefficiency is technical or political in the models of the thesis, the cost effect invariably favors price instrument—environmental tax—in the instrument choice.

Traditionally, an emission quota is modeled as fully binding. The thesis presents policy implementations in which firms' discrete choices affect the size of the emissions quota. The non-binding quota is labeled as the volume effect. As the emissions follow the quota, the volume effect makes the aggregate damage of emissions uncertain under the quantity regime. Interestingly, mainly because non-binding quota reduces the cost of cutting emissions, the thesis shows that the volume effect may also favor the quantity instrument. Notably, Marc J. Roberts and Michael Spence produced the same finding in their classical study of instrument choice in 1976, so the volume effect can be thought of as a new interpretation of their original idea.

TIIVISTELMÄ

Väitöskirjassani tutkin päästövähennystavoitteita ja päästöjen sääntelykeinojen valintaa. Väitöskirja koostuu teoreettisista tutkimuksista, joissa käytän ja laajennan Martin Weitzmanin klassikkomallia vuodelta 1974. Weitzman jakaa tutkimuksessaan sääntelykeinot hinta- ja määräkeinoihin; säätelijä määrittää joko saasteen yksikköhinnan (hintainstrumentti) tai määräkiintiön suuruuden (määräinstrumentti). Hän osoittaa, että valinta hinta- ja määräinstrumentin välillä on merkittävä. Epäsymmetrinen informaatio säätelijän ja säädeltävien välillä luo kustannuksia, joiden suuruus riippuu valitusta instrumentista. Väitöskirjassa Weitzmanin kehikkoa laajennetaan ottamalla huomioon myös diskreetit valinnat, subventiot ja kustannustehottomuus. Väitöskirjan tutkimukset osoittavat, että laajennukset vaikuttavat voimakkaasti valintaan hinta- ja määräinstrumentin välillä.

Diskreetillä valinnalla tarkoitetaan isoa investointihanketta, jossa yhdellä päätöksellä ja suurella rahallisella panostuksella saadaan aikaan huomattava päästövähennys. Yritystä subventoidaan, jos sen ei tarvitse maksaa jokaisesta tuottamastaan päästöyksiköstä. Väitöskirjassa osoitetaan, kuinka saastuttavien yritysten diskreettien valintojen subventointi aiheuttaa päästöleikkausten kustannusten allokoitumisen tehottomasti yritysten kesken. Huomionarvoista on, että työssä tutkitut sääntelymenetelmät ovat tehottomuuden mahdollinen lähde. Sekä ympäristövero (hintainstrumentti) että kaupattavien päästölupien järjestelmä (määräinstrumentti) sisältävät päästöihin sidottuja maksuja.

Väitöskirjassa päädytään tarkastelemaan kahta eri tukipolitiikkaa ja niistä kumpuavaa kustannustehottomuutta. Sanotaan, että tehottomuus voi olla luonteeltaan poliittista tai teknistä. Poliittinen tehottomuus liittyy erilaisten sidosryhmien vaikutusvaltaan. Toisin kuin säätelijällä, sidosryhmien tavoitteena ei ole taloudellisen tehokkuuden edistäminen. Säädeltävien yritysten tavoitteena on ennemmin verovähennysten tavoittelu, kun taas ympäristöliikkeet mieluusti edistävät vihreän teknologian tukitoimia.

Tekninen tehottomuus on edellistä hienovaraisempi käsite. Se liittyy ristiriitaan, joka vallitsee saastuttavan tuotantoalan monimutkaisuuden ja sitä säätelevien instrumenttien yksinkertaisuuden välillä. Väitöskirjassa kuvatussa monimutkaisuudessa on kyse teknologisesta ulkoisvaikutuksesta eli kasaantumisvaikutuksesta: mitä enemmän yritykset siirtyvät käyttämään vihreää teknologiaa, sitä edullisemmaksi sen käyttö muodostuu. Väitöskirjan analyysin mukaan tehokkuus vaatii vihreän teknologian subventointia, mutta epäsymmetrisen informaation vuoksi toteutus tarvitsee hyvin monimutkaisia instrumentteja.

Väitöskirja osoittaa, että mallilaajennukset kehittävät Weitzmanin alkuperäistä näkemystä instrumenttien valinnasta. Mallilaajennusten vaikutukset voidaan tiivistää kolmeen uuteen ilmiöön: lähtökohtailmiöön (base effect), kustannusilmiöön (cost effect) ja määräilmiöön (volume effect). Väitöskirjassa myös osoitetaan, että näiden ilmiöiden yhteisvaikutus riippuu politiikka- ja toimialakohtaisista tekijöistä.

Lähtökohtailmiö liittyy sellaisiin säädeltyjen toimialojen kokonaiskustannusten muutoksiin, joihin ei liity tehottomuutta. Tutkimus osoittaa kaksi lähdettä ilmiölle: vapaaehtoisten päästöleikkausten ohjelma ja positiivisten ulkoisvaikutusten olemassaolo. Näiden oletetaan alentavan päästöjen leikkaamisen kustannuksia. Väitöskirjassa esitetään, kuinka leikkauskustannusten laskiessa sääntelijä keskittyy enenevässä määrin sääntelyn ympäristövaikutuksiin ja siten määräsääntelyyn päästöjen kokonaiskiintiön kautta. Väitöskirjan lähtökohtailmiöt suosivatkin aina määräsääntelyä.

Kustannusilmiön taustalla on subventoinnin aiheuttama tehottomuus säädellyillä toimialoilla. Väitöskirja osoittaa, että kustannusilmiön rakenne riippuu siitä, onko tehottomuus luonteeltaan poliittista vai teknistä. Toisaalta väitöskirjassa tutkitut tapaukset osoittavat, että kustannusilmiö suosii hintainstrumenttia (ympäristöveroa) poikkeuksetta.

Päästökiintiö oletetaan teoreettisissa tutkimuksissa yleensä sitovaksi. Väitöskirjan malleissa esitetään ympäristöpolitiikan toteutuksia, joissa yritysten diskreetit valinnat aiheuttavat lupakiintiön vaihtelua. Tätä kutsutaan määräilmiöksi. Kun kokonaispäästöt seuraavat kiintiötä, määräilmiö muuttaa määräsääntelyssä päästöhaitat arvaamattomiksi. Väitöskirjani kuitenkin osoittaa, että tämä hallinnan menetys ei välttämättä vahingoita määräsääntelyä, sillä menetyksen vastapainoksi yritysten leikkauskustannukset laskevat. Ilmiö ei ole sinällään uusi, sillä jo Marc Roberts ja Michael Spence vuonna 1976 hyödynsivät heiluvaa kokonaiskiintiötä omassa sääntelyehdotuksessaan. Väitöskirjassa tämä löydös saa uuden tulkinnan määräilmiössä.

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INTRODUCTION

"I got it wrong on climate change —it's far, far worse."
—Nicholas Stern in *The Guardian* on 26 January 2013

0.1 The Research Problem

0.1.1 Market Failures and Externalities

In economics, the birth of the pollution problem is often described by the concept of market failure. For example, *The Stern Review* (Stern, Peters, Bakhshi, Bowen, Cameron, Catovsky, ... , & Edmonson [76], p.xviii) calls climate change the greatest market failure the world has ever seen.[1] The core of the problem is seen to lie in production and consumption decisions that are not coordinated by the market (or any other institution). For example, the burning of coal produces both electricity and carbon dioxide. There have been electricity markets for ages, but no corresponding carbon dioxide markets and no price for carbon dioxide. As far as production of carbon dioxide has real effects in society, zero price implies too much production and consumption of it.[2] The question of general interest is then how much electricity and carbon should we have, what type of institutions could or should be established for coordination, and how do these institutions behave and compare in society.

We will focus on the pollution problem, where the damage results from the activity of several independent polluting firms. The United States' Acid Rain Program and the EU Emissions Trading system (EU ETS) are two major attempts to create emission markets and thus prices for them. The Acid Rain Program was initiated by

[1] See also Stern and Taylor [77] and Tol and Yohe [81].

[2] Tietenberg and Lewis ([80], Chapter 2) discuss the market failure in more detail. Agnar Sandmos's book " *Economics Evolving: A History of Economic Thought*" [64] offers a broad perspective on the subject.

Title IV of the 1990 Clean Air Act Amendments and the target was electric utility emissions of sulfur dioxide (SO_2), the major precursor of acid rain. The EU ETS, created in 2005, is the EU's key tool for reducing greenhouse gas emissions. Both policy implementations work on the "cap and trade" principle. A cap is set on the total amount of emissions that can be emitted by installations covered by the system. Within the cap, companies receive or buy emission allowances that they can trade with one another. Both implementations apply free allocation in the initial allocation of permits (also called allowances). That is, regulated units receive some part of their total allowances for free. In the Acid Rain Program, the allocations were based mainly on actual fuel use during the period 1985–1987. Within the EU ETS, the transition to auctioning is taking place progressively. Some allowances will continue to be allocated for free until 2020 and beyond.[3] (European Commission [17]; U.S. Environmental Protection Agency [82]; Ellerman, Marcantonini, & Zaklan [16]; Schmalensee & Stavins [66], [67].)

One permanent theme in this thesis is the presence of an environmental externality. Externality is one type of market failure. According to Baumol and Oates [4], p.17: "*An externality is present whenever some individual's (say A's) utility or production relationships include real (that is, nonmonetary) variables, whose values are chosen by others (persons, corporations, governments) without particular attention to the effects on A's welfare*." In the case of the environment, we may think that the different parties utilize various environmental resources without particular attention to their quality. In this context, the environmental externality is negative because the presence of an externality lowers the quality of the environmental resource. We further assume that the externality has the following nature of a public bad: The downgrading of the environmental resource is reflected in the damages that depend on the sum of the individual externalities, while the benefits of production are purely private.[4] We assume that the individual members of a society are unable to agree on the level

[3]The U.S. Acid Rain Program and the EU ETS demonstrate some features of our theoretical policy framework. Note that the currently pursued carbon policies also use carbon taxes. For example, Carl and Fedor [6] recently calculated that governments around the world collect \$28.3 billion carbon revenues each year in 40 countries and 16 states or provinces. Out of this total revenue, \$6.57 billion is collected thorough cap-and-trade systems, while carbon tax systems account for \$21.7 billion.

[4]Arthur C. Pigou [57] formalized the concept of an externality, while Paul A. Samuelson [63] was the first to develop the theory of public goods. Nowadays, these concepts are an integral part of the introduction to environmental economics (see, for example, Tietenberg & Lewis [80]). A more advanced introduction to environmental policy (that shares many topics with this thesis) is Baumol and Oates [4].

of the externality on a voluntary basis. Rather, the government has the power to decide on it. Furthermore, in the hierarchal government, an environmental agency exists to have the responsibility of protecting the environment.

We have two interpretations for the existence of a negative externality. First, the producers and sufferers are separate units, so it follows that the producers have no private incentives to reduce the externality generation. Second, the producer may be a sufferer as well, but the private costs of reductions well exceed the benefits. Take as an example the individual car driver's decision in a densely populated city. Her single decision to use alternative modes of transportation has a negligible positive effect on her city air quality, but leaving her car home may cause high private costs in terms of extended travel time.

0.1.2 Payments and the Instrument Choice

This thesis aims to provide a theoretical framework for the study of instrument choice *by exploring how much and by what means societies should reduce pollution.* We assume that large-scale projects and discrete choices are the primary modes in the implementation of these reductions. Discrete choices include concerns like whether to close down an operation for good, whether to voluntarily participate in regulation, whether to modify an existing production line, or whether to invest in a new green factory. A unifying theme is the application of economic incentives in environmental regulation (Stavins [74], [75]) . We will exclusively concentrate on so-called market-based instruments in a market where a large number of firms produce and pollute. Tradable permits (known also as emissions trading) and pollution taxes are the two major market-based instruments. Combining these two themes, we show how the role of payments becomes seminal in the regulation process. In particular, as choices are large-scale and discrete, environmental payments (e.g., tax payments) have an allocative role. With small-scale and continuous choices instead, the unit price of emissions (e.g., tax rate) would allocate aggregate emissions between the regulated units. Mostly, we will study how the payments relate to the design of market-based instruments. *We ask how the size and nature of the payment burden under various regulatory projects will affect the choice between tradable permits and pollution taxes.*

In searching for the preferred instrument, we base our choice on the theoretical literature that originates from Weitzman [84]. Accordingly, the two instruments are fundamentally different as tradable permits fix the quantity from the outset while taxes fix the price. Weitzman shows how this difference, together with the asymmetry of information between the regulator and the polluting units, makes a difference between the instruments. However, in comparing the different modes, Weitzman does not explicitly pay attention to the monetary payment flows from the regulated units to the regulator. In this thesis, we complement the study of Weitzman [84] by examining the extent to which these payments affect the instrument choice.

There exists a certain type of inertia in the Weitzman framework. The regulator commits in advance not to reset the policy even though some new information may arrive. It commits to a fixed unit price in price policy or it commits to a certain number of tradable permits in quantity policy. Clearly, the commitment is a restriction in policy-making, so it will yield some costs as well. In price policy, costs arise as emissions fluctuate too much compared to the optimal level, while in quantity policy (with a fixed level of emissions), the emissions react too little. Similarly, in price policy (with a fixed price level), the emission price reacts too little, while in the quantity policy, it overreacts. Overall, the commitment either to price or quantity policy yields a simple and understandable policy framework, but the commitments will result in too extreme outcomes in the market. To study the costs of the policies, Weitzman applies a linear-quadratic framework to social welfare. In particular, uncertainty enters in a linear fashion into his model. As the quadratic framework implies linear marginal costs and benefit functions, the commitment costs (the reduction in social welfare as compared to optimal welfare) depend on these. Actually, Weitzman shows *that the instrument choice depends on the slopes of the marginal functions alone*. If the slope of the marginal damage function exceeds the slope of the marginal benefit function, the level of emissions should be fixed. Conversely, if the slope of the marginal damage function is smaller than the slope of the marginal benefit function, the price instrument with a fixed unit price is the proper instrument choice. Then, in a formal sense, this thesis aims to investigate how various payments in the policy implementations will affect this simple rule in instrument choice.

As stressed above, we will primarily concentrate on the emission reductions that the regulated units accomplish through discrete means. In other words, we study regulation, where the aggregate reductions and the corresponding regulation depend heavily on discrete choices. The distinction between continuous and discrete choices roughly corresponds to the distinction between short- and long-term choices. Over the short term, firms in the polluting industry implement emission reductions by merely adjusting the output downwards. In the long term, firms modify their production technologies to integrate their production efficiently into the new regulatory framework. Firms may engage in small-scale projects by adjusting the existing technology. In these projects, they may modify their product lines by switching to less polluting inputs. Alternatively, they may install end-of-pipe purification technologies, where detrimental contents are removed from waste discharges. Even more decisively, firms may launch large-scale investments and may replace the old polluting factories with new ones. After wholly accounting for the environmental impacts, the new green technology may produce the output much more efficiently. Finally, after every alternative has been considered, firms have the discrete mean of closing down the operation altogether.

If the changes in the operating environment are large and permanent, firms will probably question their current activity. We assume in the thesis that environmental regulation represents such a challenge *par excellence*. Influential and permanent projects can fundamentally modify the old ways of producing. Consequently, firms cannot halve aggregate emissions by halving the current production. Rather, they can halve the emissions by employing discrete and major modifications of their operations. Furthermore, we assume substantial uncertainties. The choices are discrete or large-scale projects with plenty of complexities, and the true costs and benefits become apparent only over time.

However, it is not about these discrete choices in itself but about the nature of payments that matters. In our framework, the Weitzman analysis is not refined by the mere inclusion of discrete choices but by incorporating certain types of payments into the regulation. Certainly, payments and discrete choices are connected. The payments are the main allocative mechanism in this work, and they exclusively operate through discrete choices. Apparently, as the payment burden of the current technology increases, a discrete switch into a less polluting alternative will look more attractive.[5] However, the thesis argues that the most important thing is that different payment burdens will affect the discrete choices differently, and some choice patterns have important repercussions in environmental policy.

We say that payments are neutral if they implement an efficient allocation of

[5]See the illustrative analysis in Amacher and Malik [2].

emissions among the firms in the polluting industry. Consequently, the instrument choice will follow the original Weitzman [84] rule. Alternatively, we say that the payments are non-neutral if the corresponding allocation of emissions is not efficiently distributed among polluting firms. With continuous choices, the thesis shows that allocation is efficient as long as marginal benefits of emissions are equal across the polluters, while *with discrete choices, allocation is efficient if average benefits of emissions are equal between the polluters*. Mainly, this thesis investigates the consequences of non-neutral payments that induce inefficient allocation of emissions.

0.1.3 Two Dimensions of a Subsidy

It is important to note why the policy employs non-neutral payments. One general answer is that neutral payments are infeasible to environmental agency. To understand this, we first need to concentrate on the nature of payments. In particular, we discuss what kind of payment a subsidy payment actually is. Overall, the size and the degree of conditionality are two major dimensions of a payment. The size of the payment is a positive continuous variable, while the degree of conditionality is dichotomous. A payment is conditional if the firm has to satisfy a certain condition to be eligible for it. For example, the firm may have to be an active producer to get the payment. Alternatively, the firm may have to use certain "green" technology to be eligible for the payment. A payment is said to be non-conditional if the only criteria for the payment is belonging to the current cohort of polluting firms.

According to the definition in Pezzey [54], we define the term "subsidy" as a non-zero and conditional payment. Thus, if firms or instruments are subsidized, the policy applies non-zero and conditional payments. Conversely, firms or instruments are not subsidized if the policy applies zero or non-conditional payments.

The subsidization issue arises from the very basic implementations of the tax and permit systems. In both cases, the aggregate payment scheme may be written as

$$s(e-l), \tag{1}$$

where e is the level of emissions. The scheme imposes the amount of money (the payment burden) that a single polluter pays after polluting e. The symbol s is the unit price of emissions, while $l \geq 0$ stands for a threshold. The threshold level of emissions is not charged at all. Depending on the implementation, s is either the permit price or the tax rate while l is either the level of free allowances or the level of tax-free emissions. In the thesis, the amount sl represents a subsidy to a polluting firm: i) if a polluting firm does not pay for every pollution unit that it produces (i.e., $l > 0$) or ii) it is a conditional payment. More generally, we use the concept payment rule $S(s)$, so $S(s) = sl$ in Equation (1).[6]

A unifying theme in the thesis is that a payment rule always depends on the unit price of emissions. Another theme concerns the relationship between payments and instruments in so-called restricted regimes. In these regimes, the regulatory agency cannot choose the sizes of the thresholds but rather can take them as exogenously given. We cannot find any reason why the given thresholds should differ between the tax and permit systems. Thus, we assume that the emission thresholds are always equal in different systems. However, the payments between the instruments will differ as the tax and permit prices will usually differ.

The thesis concentrates entirely on subsidized instruments. Within non-subsidized outcomes, we note that the celebrated "polluter pays" principle sets $l = 0$. For example, regarding the distribution of initial permit allocation, the principle advises to auction them off entirely. This may raise strong resistance among the influential polluting industry and may finally become infeasible in practice. The main issue is that it will dramatically increase the cost of polluting units overnight. It is likely that an alternative, such as a negotiated solution, supports partial auctioning, so the policy ends up subsidizing the polluting industry. In fact, in reviewing the United States' Acid Rain Program, Schmalensee and Stavins [67] write about substantial political value of the free allowances. Free allocation built important support for the regulation by addressing differential economic concerns (see also Joskow & Schmalensee [27] and Ellerman, Joskow, Schmalensee, Bailey, & Montero [13], Chapter 3).

On the other hand, a non-conditional and positive payment represents a very special type of payment profile. Specifically, non-conditionality means that the environmental agency has to commit to positive payments for the current polluting firms forever. This type of perpetual commitment sounds expensive and difficult to justify, so it may well become infeasible.[7] Similarly, there may well exist needs

[6] We will encounter a specific payment rule in Chapter 3 that does not depend on threshold l at all.

[7] One may get the impression that the resource ownership should be distributed as well. Consequently, if societies want to reduce emissions in the future, they cannot do that without hurting ownership rights. However, we think that there is no intrinsic reason to tie the payment to ownership. That is, the payments can be paid as before even though the political process cuts emissions. About this subject, see also footnote 28.

to support the transition towards emerging green technology. However, as we later show, the principle of non-conditionality rules that dirty technologies should also be similarly supported. This last requirement, not least for budgetary reasons, may become infeasible as well.

Non-subsidized instruments may be difficult to implement in practice, but they are theoretically interesting. This is because the payments of non-subsidized instruments do not affect the choices of the polluting firms. More precisely, they do not affect the choices between different discrete alternatives. To understand this assertion, recall the opportunity cost calculations that determine firms' choices. Because of non-conditionality, the payments S will totally vanish from the calculations.[8] Intuitively, they disappear as the firms get the same payment independent of their choices. Note that the zero payment rule represents just a special case of non-conditionality.

0.1.4 Subsidized Instrument Choice

The unifying theme in the thesis is that environmental policy applies subsidies. Our analysis rests on the general idea that, once the subsidization is allowed, then the door is open for non-neutral outcomes. In the thesis, non-neutrality means inefficiency in emission allocation, and this will fundamentally influence the environmental policy. In particular, we show that it affects the instrument choice, in which the environmental agency not only decides how much to reduce pollution by but also decides the means of reduction. The choice is between environmental taxes (prices) and tradable permits (quantities). We will capture the influences of subsidization by augmenting the original concept of comparative advantage (Weitzman [84]). We present and discuss the augmentation in Subsection 0.4.2.

We will develop this basic theme in three main chapters. All chapters represent different types of approaches in subsidization. Our first approach (Chapter 1) studies purely political motives behind the subsidization. The environmental agency sees no welfare motives for subsidization. One influential motive to subsidize relates to the increased cost burden that the firms in the regulated industry have to bear. These firms are subsidized to mitigate the consequences of the change. Another politically colored motive concerns the principle that green investments should be subsidized.

[8] These types of opportunity cost calculations are illustrated in Pezzey [54].

In our second approach (Chapter 2), however, the subsidies are vital in correcting the weaknesses of existing regulation. The specific concern is an imperfect participation, whereby the mandatory regulation covers only part of the relevant polluters. Voluntary participation provisions can facilitate the impacts of imperfect participation (as happened in the Acid Rain Program).[9] We show that implementation of such a provision means heavy subsidization of non-affected firms. In our third approach (Chapter 3), subsidization is required because of positive externalities. The investments in green technology in the parent sector is assumed to enhance the productivity of green technology in another sector. We argue that without proper subsidization in the parent sector, firms do not properly invest in green technology. Note that the positive and negative externalities interact. Clean investments have a central role in alleviating the negative externality problem—the pollution.

In theory, separate polluting sectors achieve the most efficient way of using emissions if they are allowed to trade emissions among themselves (Stavins [74], [75]). We will study real-life implementations in which the efficiency property of market-based instruments is being disturbed. Our general goal is to study the question: What does the efficiency loss mean in the Weitzman [84] framework.[10] The Weitzman framework is essentially a second-best case of regulation under uncertainty. It asks whether the environmental agency should commit to price instrument or quantity instrument, *when the agency is forced to make and keep the commitment.* Consequently, the agency cannot reset the policy parameters once new information arrives.

The second-best framework ignores distributional considerations. On the contrary, we are particularly focused on the distributional properties. Regarding the various influences of instrument choice, we gather them under three broad concepts: cost, volume, and base effects. Cost effect is a pure measure of inefficiency in instrument choice. We identify two types of regulatory circumstances where cost effects emerge. In Chapters 1 and 2, the efficiency is disturbed. That is, the environmental agency monitors how efficiency is being ruined by subsidization. In Chapter 3, instead, the agency intentionally aims for efficiency by using subsidies. However, the agency cannot retain efficiency by the simple linear instruments of the regulatory toolbox. To retain efficiency, we show how the positive spillover among the

[9] Title IV's Voluntary Compliance Program is reviewed in Ellerman *et al.* ([13], Chapter 8).

[10] Other factors than subsidization may influence efficiency. For instance, in the implementations of market-based instruments, various administrative and transaction costs (Polinsky & Shavell [58]; Stavins [72]) emerge that drive the allocation away from efficiency. Alternatively, the monitoring and enforcement of market-based regulation may well distort the efficiency property (Malik [37]).

polluting industry calls for more complex state-contingent payment rules or active updating of the original payments. Inefficiency follows as we assume these are infeasible. Overall, we always end up studying inefficient outcomes. Whatever the reason is behind inefficiency we find a qualitative similarity between the implementations. That is, the cost effect will always favor the price instrument.

The volume effect relates to emission quotas. Traditionally, the quota is a fixed level of emissions that equals the aggregate number of permits in the market. In the thesis, the possibility that the quota (and emissions) fluctuates relates fundamentally to quantity implementations that will apply emission thresholds (see Equation (1) above). These thresholds are influential as they efficiently account for some part of the total emission quota. We show how the agency may control either the aggregate number of permits or aggregate number of auctioned permits in the threshold implementations. In the last of these cases, the quota and the corresponding aggregate emission level will fluctuate. However, we show that a fluctuating quota is not automatically detrimental to quantity implementation. It may be beneficial as well. After all, the absolute strictness of the emission quota induces costs in the original Weitzman instrument choice. Especially in Chapter 1, we study circumstances, where the volume effect does benefit the quantity instrument. We also learn that the volume effect will flourish only when the implementation uses inefficient thresholds.[11]

A base effect reflects the effect on instrument choice that the switch from one efficient implementation to another creates. This concept records the consequences on the benefit side. In Chapter 2, the base effect follows as the scope of the regulation increases. Specifically, if the number of cost-beneficial projects is increasing alongside the increasing scope, the slope of the marginal benefit function will decrease. On the other hand, in Chapter 3, the base effect arises as the knowledge spillover reduces the slope of the marginal benefit function. Overall, we show that a base effect influences instrument choice in a traditional way. The decrease in the slope of the marginal benefit function will favor the quantity regulation. This principle reflects the view that a decrease in the costs of regulation allows the regulation to emphasize the emission damages, and consequently, to emphasize the stability of emissions.

In summary, we study how different payments control discrete choices in a regulated industry. Referring to our discussion above, the payment consists of two

[11] However, the converse is not true. We discuss how the policy can eliminate the volume effect but it cannot eliminate the cost effect.

parts: se and $S(s)$. The first of these, se, is the price of emissions times the amount of emissions, and this part of the payment can be loosely interpreted as a factor payment.[12] The second part, $S(s)$, is a subsidy that offers flexibility in setting the payments. We then assume that environmental policy is implemented with market-based instruments and that the subsidy depends on the unit price. A key feature in environmental taxation is the fixed unit price of emissions. Then, even though there are uncertainties in the polluting industry that will shift the marginal benefit functions, the payment remains intact. Alternatively, the regulation is implemented by tradable permits. In this case, uncertainties are transformed into permit price, so the payments become variable as well. Most importantly, there are subsidies that import inefficiencies into the permit markets. The inefficiencies are reflected in the relation between taxes and tradable permits. We then say that inefficiencies affect instrument choice.

0.2 Modeling Choices

0.2.1 Regulation of an Externality

There are similarities as well as differences between taxes and tradable permits (Keohane [31]; Keohane, Revesz, & Stavins [32]; Ellerman [14]; Carl & Fedor [6]; Pezzey [54]). To a certain extent, they are equal instruments, but in reality, differences exists, so comparative study is needed. The major similarity between instruments is that, in reducing the overall pollution, both instruments set a price for pollution. The idea is that costly pollution will induce firms to reduce pollution. Presumably, when firms learn that a ton of carbon has a price, they will try to cut back on producing pollution, convert the product line to a less polluting one, or build a new factory. Some firms will cease to operate altogether. One major difference between the instruments is that they implement the price differently. Environmental taxation is a price instrument that sets the price directly, while a tradable permits system is a quantity instrument that sets the price indirectly. As far as the ultimate target of society lies in a certain pollution reduction, we can say that the instruments achieve this target differently. A quantity instrument achieves it directly by setting a binding cap on the emissions, while the tax instrument achieves it indirectly by making the

[12]For such an interpretation, see Spulber [71].

pollution costly. Weitzman [84], in particular, emphasizes this difference. He bases his comparative study on the idea that uncertainty gives rise to a specific choice problem. The regulator has to choose whether it will lose control of the permit price or of the taxed emissions. Weitzman is able to derive a very concise and intuitive result for the proper societal instrument choice.

The commitment of the environmental agency is the core of the Weitzman [84] analysis. Even though the amendments in the regulated industry give reasons to re-evaluate the policy in the future, the agency will not do that. This may reflect the nature of regulatory services. If the regulatory agency can react cheaply and quickly to changes in the regulatory environment, then the incentives for the commitment are low and the Weitzman analysis is not helpful. On the contrary, if investments "... *that will be made in any pollution control program will take several years to plan and complete and will be largely irreversible once in place*" (Roberts & Spence [62], p. 193), then the Weitzman approach is appropriate. In what follows, we will concentrate on the Weitzman type "once-and-for-all" policies, in which the policy is set and not revised for a sufficiently long period.

We assume that the polluting units emit the same type of pollution, specifically, that emissions are homogenous in nature. We then require that pollution damages depend entirely on the level of aggregate emissions, not on the distribution of emissions between the firms. In their own way, the United States' Acid Rain Program and the EU ETS satisfy this condition. For example, the atmospheric lifetime of carbon dioxide (CO_2) is long compared to the timescales of global atmospheric mixing (European Environmental Agency [12]). Over time, CO_2 emissions spread evenly into the atmosphere. On the other hand, even though sulfur dioxide (SO_2) emissions do not mix perfectly, the Acid Rain Program treats them as if they do. There were some concerns about "hot spots" (i.e., significant local extreme concentrations), but these concerns did not materialize. (Schmalensee & Stavins [66]; Ellerman *et al.* [13].)

Externalities are not always negative; they may be positive as well. In fact, in Chapter 3, regulation is studied in a situation in which both negative and positive externalities exist simultaneously. Even though this is a topic of general interest, we treat the positive externality explicitly as a knowledge spillover.[13] Knowledge spillover is a special kind of externality that exists whenever an additional applica-

[13] Within this context, the diffusion of technology (as examined by Griliches [21], [22]) provides an appropriate background. Regarding the simultaneous presence of knowledge spillover and an environmental externality, see Jaffe, Newell, and Stavins [25].

tion of a certain technology becomes easier as the number of applications increases. This process qualifies as an externality, as the beneficial side effects are external to the intended private profit-maximizing choices. Furthermore, there is an additional link between the different types of externalities in Chapter 3, as the transition towards green production in a leading sector benefits the follower sectors. We use the term "green production" (as opposed to "brown production") throughout the thesis to refer to a type of technology that has a relatively low negative impact on the environment.

0.2.2 The Polluting Industry

We have two polluting sectors that together form the polluting industry. The individual polluters are called firms or (polluting) units. We use the words interchangeably to refer to the subjects of regulation. The polluting units may use different production technologies.[14] The choices of the polluting units are taken as discrete in this thesis. A unit decides whether to produce, whether to update the existing technology, whether to use brown (polluting) technology or green (clean) technology, or whether to participate voluntarily in an environmental program. Furthermore, if a unit produces, it will use its entire capacity. Within a polluting sector, there may exist at most two technologies. Given unit λ that uses technology j within sector i, its profit is

$$\Pi_i^j(\lambda) = B_i^j(\lambda) - s(\alpha_i^j - l_i^j) \tag{2}$$

with

$$B_i^j(\lambda) = b_i^j + \theta_i^j - c_i^j \lambda, \tag{3}$$

where $i = 0, 1$ and $j = b, g$. We denote the benefits as $B_i^j(\lambda)$, where b_i^j and c_i^j are positive constants and θ_i^j is a random variable. We assume additive uncertainty, where $E(\theta_i^j) = 0$ and every covariance between the uncertainty variables equals zero. Furthermore, in the profit equation, the emission factors are assumed constant within sectors and technologies and are denoted by α_i^j. Thus, the production in sector i by technology j produces emissions α_i^j. We have $\alpha_i^b > \alpha_i^g$, so there exists a green

[14]For example, a power company may consist of several power plants. It is the plant, not the company, that is the ultimate regulatory subject.

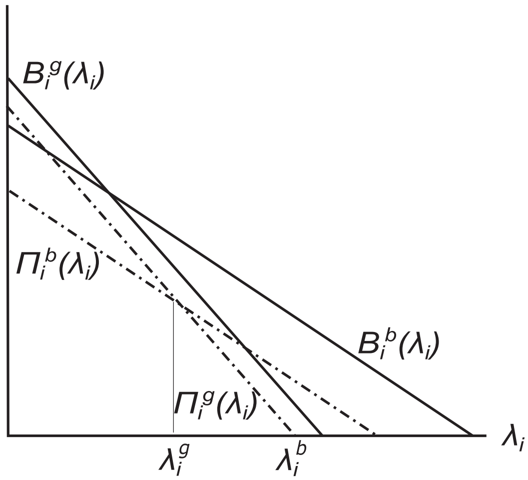

Figure 1 A Polluting Industry

alternative to brown production. The unit price of emissions is denoted by s and l_i^j is the subsidy threshold. This notation allows both sector- and technology-specific thresholds. Whenever there exists only one technology in sector i, we just skip the superscript j.

Figure 1 illustrates polluting sector i when $j=2$. The profit lines (Π_i^j) are drawn by non-solid lines and the benefits (B_i^j) by solid lines. We assume (without loss of generality) that $b_i^g > b_i^b$ and $c_i^g > c_i^b$. In Figure 1, this assumption means that units at the low end of the distribution use green technology while units at the high end use brown technology. In particular, there are two cut-off units within an industry. First, unit λ_i^b satisfies

$$\Pi_i^b(\lambda_i^b) = 0$$

in industry i. The unit is indifferent between producing and exiting the market. Second, there is unit λ_i^g in industry i that satisfies

$$\Pi_i^g(\lambda_i^g) = \Pi_i^b(\lambda_i^g).$$

This particular unit absolutely produces, but it is indifferent between green and brown technology.

We consider the units as negligible (McKitrick & Collinge [41]; Spulber [71]). By definition, an entry of an additional unit has negligible effects on marginal social damages. From a purely practical point of view, the units are small enough so that differentiation and integration are plausible methods. Furthermore, without loss of generality, we may take the unit λ as an integer. Thus, λ_i^b is the number of polluting units in sector i. As λ_i^g is the number of units that use green technology, then $\lambda_i^b - \lambda_i^g$ is the number of units that use brown technology in sector i. Note also in Figure 1 how the profit is a piecewise linear function of λ, where the size of the kink is equal to the number of green units.

The profit function is generalized in Chapter 3, where we assume the presence of both positive and negative externalities. As for the positive one, we assume the presence of knowledge spillovers. This means that an additional application of a certain technology becomes easier in a particular sector, as the number of applications increase in another sector. More formally, sector i's benefits read as

$$B_i^j(\lambda) = b_i^j + \theta_i^j - c_i^j \lambda + \phi^j \lambda_k^j, \tag{4}$$

where $i \neq k$. We say that sector i is the externality-receiving sector while sector k is the externality-generating sector. We see how the number of units in sector k that apply technology j (λ_k^j) affects the benefits in sector i that are derived from technology j. As we assume that the externality is positive, the parameter ϕ^j is strictly positive. If $\phi^j = 0$, then we are back in the basic formula of Equation (3). The externality is assumed to flow only in one direction, so the choices in sector i do not affect profits in sector k. Furthermore, we will assume $\phi^b = 0$ and $\phi^g = \phi > 0$, so the positive spillover operates only within green technology.

These assumptions together allow the simultaneous determination of four choice variables. However, in the discrete choice models to come, we will somewhat restrict the generality of the analysis as we restrict the number of choices to two. We do that to keep our analysis tractable. We also want to emphasize that factor sl_i^j is a sum of money (a payment), not a unit price (euros per ton of emissions). This makes a difference, as one could argue that our model depicts a short-run model with two firms in a polluting industry. In a short-run firm interpretation, factor sl_i^j disappears from the profit calculations (Equation (2)). This happens as payments do not matter in the short-run. We conclude that the presence of payments is a key element in our analysis.

0.2.3 Elements of the Regulatory Design

Throughout the thesis, an environmental externality exists and is calling for internalization. This task has been assigned to the environmental agency.[15] The agency is welfaristic because its sole objective is maximization of social welfare, the difference between the benefits and damages of emissions.[16] We assume that informational and distributional issues constrain the working of the agency.

We will follow Weitzman [84] by assuming a particular type of asymmetrical information between the regulatory agency and the regulated units. In the implementation stage, the regulatory agency knows the values of b_i^j and c_i^j in the benefit function (Equation (3) above). It also knows that $E(\theta_i^j) = 0$ and that it can with certainty identify the sector in which each unit is located. In contrast, every unit knows the values of θ_i^j with certainty. Furthermore, we will make an additional assumption that the agency finds some information too costly to acquire. In practice, the regulatory agency learns the values of b_i^j and c_i^j for free, but it cannot link a unit to the correct type within a sector. It then follows that the agency knows the type distribution but it does not know the profit of any particular firm.

We also assume that the regulatory agency will apply so-called market-based instruments. Market-based instruments "*encourage behavior through market signals rather than through explicit directives regarding pollution control levels or methods*" (Stavins [75], p. 9). In this context, "the explicit directives" refer to firm-specific regulations or to uniform regulation. For example, if the regulated choice concerns the choice of the applied technology, firm-specific regulation means that the agency determines the sufficient technology on a case-by-case basis. Under uniform regulation, instead, every operating firm must apply the same technology. On the contrary, if market-based instruments are applied, each regulated unit can decide what technology to use. The units comply as long as they pay the tax or have a sufficient number of emission permits.

In the absence of regulation, there are too many polluting units in operation. The use of a market-based instrument is an attempt to solve this problem. Consequently,

[15] Occasionally, we use the term "the regulator" or "the regulatory agency".

[16] Throughout the thesis, we will mainly operate in terms of emissions, not in terms of emission reductions. However, these two approaches are fully equivalent (see the textbook presentation in Tietenberg and Lewis ([80], Chapter 14)). Occasionally, we talk about emission reductions and costs of emission reductions. For example, the term "cost inefficiency" means that aggregate emissions reduction is inefficiently organized between polluting firms.

as the policy set an environmental payment (equal to $s(\alpha_i^j - l_i^j)$ in Equation (2)) to polluting units, it will eliminate the most unproductive units from the market. These units may switch toward greener production or shut down the entire production.

In general, factor $s l_i^j$ in the profit function (Equation (2)) is called a payment rule. The rule is either a subsidy or a property rights payment. According to Pezzey [54], a payment rule is a subsidy payment when a firm gets it only if it produces output. The payment rule is a property rights payment when a firm gets it whether or not it produces.[17] We assume that property rights payments are infeasible and concentrate on the subsidy implementation. Furthermore, thresholds can be either inframarginal ($l_i^j < \alpha_i^j$) or supramarginal ($l_i^j > \alpha_i^j$) (Pezzey [54]). If thresholds are inframarginal (supramarginal) in sector i among the units that apply technology j, then these units are demanders (suppliers) in the permit markets.

We further assume that the agency can apply technology- and sector-specific subsidies. In a sense, we broaden the definition of Pezzey [54] as we include conditions beyond the production of output. For instance, policy $l_i^b = 0$ and $l_i^g > 0$ mean that only green technology is subsidized in sector i. Payment $s l_i^g$ is a subsidy, so it is paid to a unit only if it produces by using green technology. Note also that all the rules so far explicitly depend on the thresholds. In Chapter 3, we will study another type of regime. In particular, we will apply scheme

$$se - S(s), \tag{5}$$

where

$$S(s) = \Gamma + \Gamma_s s. \tag{6}$$

In the payment rule, $\Gamma > 0$ and $\Gamma_s > 0$ are fixed parameters and s is the unit price of emissions. The values of Γ and Γ_s are determined as part of the policy optimization, and they do not depend on the thresholds. Subsidy payment $S(s)$ is needed in Chapter 3 to internalize the positive externality. Overall, an important part of our analysis is the assumption that the subsidy depends on the unit price of emissions.[18]

We say that the policy is efficient if the distribution of total emissions is efficient among the regulated units. An efficient allocation then maximizes the aggregate

[17] The property rights payment is given only to existing firms. Pezzey [54] also requires that the property rights have have "good characteristics in all six dimensions" as presented by Devlin and Grafton [11].

[18] Yet, we present a case in Chapter 1 in which a fixed subsidy (the same subsidy under prices and quantities) affects the policy strictness but does not affect the instrument choice.

profits given a fixed level of emissions. An optimal policy instead maximizes overall social welfare, so the policy deliberately accounts for the damages of emissions. While optimal policy is always efficient, the reverse is not true. If some level of emissions is not efficient, the social welfare will not decrease, if the emissions distribution becomes efficient. On the other hand, the emissions may be efficiently distributed, but the policy is not optimal. In Chapter 2, we study voluntary participation, where the industry is divided into affected and non-affected firms. We assume that the emission distribution among the affected firms is efficient. However, the social welfare may increase if the agency can successfully incorporate non-affected firms into regulation.

Regarding the damages of emissions, we assume that

$$D(e) = \varepsilon e + \frac{d}{2} e^2,$$

where $d > 0$, ε is a random variable (with $E(\varepsilon) = 0$) and e is the aggregate level of emissions. The individual emissions are homogenous in nature, so only the aggregate level of emissions matters. Furthermore, we assume that

$$E\left(\theta_i^j \varepsilon\right) = 0,$$

where $i = 0, 1$ and $j = b, g$. Like in the original Weitzman analysis, this assumption means that the damage uncertainty does not affect the instrument choice.[19] Thus, without loss of generality, we will set $\varepsilon \equiv 0$ from the outset.

0.3 Discussion of the Modeling Choices

0.3.1 Information

Our approach expands the study of instrument choice under uncertainty (Weitzman [84]) towards cases of subsidization.[20] The subject is important because there is a growing interest in market-based instruments and because subsidization of firms is widely applied in real-life policy implementations. Furthermore, even though in-

[19]We will show in Chapter 1 more closely how this result holds in our analyses.

[20]Weitzman's classical study was published about 45 years ago. Martin L. Weitzman continued his research in this specific field of environmental economics throughout his long career (Mideksa & Weitzman [44]; Wagner & Weitzman [83]; Weitzman [85], [86], [87], [88], [89]).

strument choice under uncertainty has a relatively long history and is still under intense research, our particular subject has not received much attention.

Throughout the thesis, we apply a framework that includes asymmetric information. First, information is asymmetric because information is costly. If information is costly, then the agency does not know the profits of any particular unit. Rather, it has only general information about the sectors. In particular, the applications of unit-specific policies become non-feasible mainly because the information is costly (for example, see Montero [47]). This feature has consequences on the choices of policy instruments. Specifically, the "command and control" type of policies become difficult to implement. As an example, assume that the units have access to two types of technologies: brown and green. Assume further (like we do in Chapters 2 and 3) that the regulatory outcome is a partial technological replacement, where both types of technologies should produce side by side within the sector. As the information is too costly to acquire, the agency cannot implement unit-specific technology standards in a straightforward manner. With limited information about the unit-specific profitabilities, it cannot pick out the relevant units from the mass of polluters. Instead, if the implementation of the policy target is based on market-based policies, the regulation works differently. Importantly, there is no unit-specificity in the payment rule.[21] As another example, we may think of a targeted subsidy program in which the policy should target the subsidies to some specific firms. There may exists political pressure to support some firms against the threat of the closure. Thus, costly information most certainly complicates the implementation of targeted policies.[22]

We deliberately apply the term "costly information" as we reckon that there is another source of asymmetric information: uncertainty.[23] We follow Weitzman [84] as we assume that uncertainty operates in a leader–follower relationship. At the time the agency initiates the regulation, there exists commonly shared uncertainty about the profitabilities of the regulated units. Weitzman assumes that the polluting

[21] In our framework, the profit function in Equation (2) shows how the payments may depend on sector i and on technology j through threshold l_i^j, but the payments do not depend on the type of unit λ_i^j.

[22] In the literature of instrument choice, the superiority of market-based instruments over non-tradable quotas has been questioned (Malueg [39]; Sartzetakis [65]; Kato [28], [29]). However, this thesis does not supplement this line of research as it compares only different market-based instruments.

[23] In practice, asymmetric information allows the regulator to derive the aggregate abatement cost function almost perfectly. She only lacks the value of one parameter. For reference, asymmetric information is the same as in Montero [49].

units may flexibly react to changes in the profitabilities while the regulatory agency is stuck to its original policy.[24] We will follow this interpretation closely. Note also that we could have incorporated uncertainty into decision-making of the regulated units as well. We will review such an approach below as we discuss the sequential decision-making in Shinkuma and Sugeta [68]. In sequential decision-making, regulated units reconsider their initial choice after the original information has become more accurate. Furthermore, in a world of discrete choices with sequential decision-making, another complexity will arise if the choices are irreversible. For example, Baldursson and von der Fehr [3] study instrument choice in such a framework.

Furthermore, we will follow Weitzman [84] by concentrating on simple instruments. Weitzman ([84], p.481) writes: "*The reason we specialize to price and quantity signals is that these are two simple messages, easily comprehended, traditionally employed, and frequently contrasted.*".[25] Under our simple policy, every polluter in industry i using technology j faces a payment denoted by $S_i^j(s)$. Even though payment rule $S_i^j(s)$ is missing in Weitzman, it can well be considered as a simple message. In particular, the payment rule is always a linear rule. The parameters are fixed in advance and, as far as the commitment of the environmental agency is concerned, they truly stay unchanged over the course of regulation.

However, concerning the threshold implementation of rule $S_i^j(s)$, there emerges a further question of commitment. It arises as the thresholds efficiently account for part of the total emission quota. In an extreme implementation, the entire permit endowment is grandfathered in as emission thresholds. We do not follow this type of grandfathering policy in the thesis, but rather preserve a number of permits as an additional regulatory tool in the permit policy. As for the application of the tool, we present two alternatives: The agency can commit to the number of aggregate permits or to the number of aggregate auctioned permits. We discuss the choice between these alternatives below as we deal with sterilized and non-sterilized permit systems. This choice is not covered in Weitzman [84] but rather emerges because of the subsidization issue.

Admittedly, the instrument study is somewhat restricted, as the theory provides more sophisticated instruments than the simple instruments studied here. Briefly, the fact is that the sophisticated instruments promise higher expected welfare than

[24]Strictly speaking, the analysis is static in nature. A truly dynamic analysis would consider the regulation in a multi-period framework.

[25]The term "signal" is replaced by the term "instrument" in the thesis.

the simple instruments alone. Montero [50] reviews the literature on sophisticated instruments and then proposes "a simple auction mechanism" for efficient implementation.[26]

0.3.2 Industry Structure

Our ultimate goal is to study the role of subsidization in environmental policy. We will accomplish this by assuming that the units in a polluting industry make only discrete choices. Furthermore, we assume that units within the same polluting sector i and technology j have the same capacity size (and have identical emissions) but they differ in their effectiveness in applying their capacity. Naturally, the firm characteristics may also affect the size of the capacity that firms will build and affect the level of pollution that they will emit. However, even if we include endogenous emissions into the model, the influence of subsidization within discrete choices remains a topical issue (see, for example, Amacher & Malik [2]). The endogeneity will rather bring new elements into the analysis and will certainly complicate it. Under current assumptions, we can solve our models analytically, so the analysis provides opportunities for building intuition. Second, regarding our concentration only on discrete choices, we note how subsidization has an important influence on them. Discrete choices determine the number of units in any sector with a certain type of technology. Consequently, our assumption offers a novel opportunity to study *ex-post* changes in the numbers of polluting units and the influence of the changes on instrument choice. To our knowledge, nobody has previously studied the instrument choices and discrete choices together in a similar way. Third, uncertainty is a central ingredient in our thesis. We claim that the uncertainties will most likely flourish in discrete choices. These are major choices with plenty of complexities between different alternatives.

We divide the polluting units into two polluting sectors that together form a polluting industry. In permit implementations, we assume that the agency auctions off some amount of the permit's endowment. As we noted above, permit auctions are taking place progressively in the European Trading System. In the United States, Title IV mandates the EPA to administer annual allowance auctions. The initial role of these auctions was to provide liquidity and stimulus for the market, but the size

[26] See also Lee [36] and the review of Lewis [38].

of the activity was kept relatively low (Ellerman *et al*. [13], Chapter 7).

A branch of literature has already studied the determination of polluting industries under market-based instruments. Even though it applies the framework of competitive long-run equilibrium, the subject is relevant in this thesis as well. Pezzey's [54] review of the literature finds three competitive views and suggests a unified view.[27] He shows how property rights payments help to preserve the long-run efficiency of the market-based instruments (see also Åhman, Burtraw, Kruger, and Zetterberg [93]). Perhaps surprisingly, the reason is that they do not affect the entry-exit choices of polluting firms. They do not affect choices because firms receive the property rights payment independent of their observed choices. The use of property rights payments increases the flexibility of the regulatory design because it can separate distributional and efficiency issues from each other. In particular, property rights payments can implement the allocation of emissions efficiently for a variety of distributional objectives. In this sense, the subsidy is a poor substitute because the payments inherently depend on the choices of the firms.

Since our focus lies on the subsidized outcomes, we only briefly comment on property rights payments outcomes.[28] The basic relation between a subsidy and a property rights payment (as presented by Pezzey [54]) can be found in the thesis as well. However, as our frameworks are not always standard, the working of the property rights payments will be affected. This becomes evident at least in two cases: In Chapter 2, we will operate in a non-standard regulation, where the payments are

[27]There is "a conventional view" introduced by Baumol and Oates [4] in their textbook (Chapter 14) that finds tax thresholds problematic but the free allocation of permits non-problematic in terms of long-run efficiency. Kling and Zhao [30] represent the second view, "a less common view" that finds the free allocation of permits problematic. The third view, "a minority view" by Pezzey [55] and Farrow [18], states that both instruments achieve long-run efficiency irrespective of the applied threshold. Pezzey [54] explains the different views by the different assumptions. Differences will arise as the various approaches implement the payment rule either as a subsidy or as a property rights payment.

[28]It seems that the nature of the payments differ between the two major permit implementations: Acid Rain Program and EU ETS. Joskow and Scmamalansee [27] (p.39, footnote 4) write that "*Technically, the SO_2 allowances are not property rights, since Congress can change the number of allowances issued or do away with them altogether without raising a constitutional claim for compensation.... In all other respects, however, allowances are treated as property rights*." In these other respects, the authors remark how the authority provides easiness in transferability and confidence in durability for the allocated allowances. As a matter of fact, Ellerman [15] writes that closing facilities are able to retain the initial SO_2 endowments. In these respects, the free allocation of SO_2 allowances qualifies as a property rights payment. Ellerman [15] further notes that a closing facility forfeits future EU ETS allowances. The treatment of closures in the National Allocation Plan (NAP) is further discussed in Åhman *et al.* [93] and more recently in Woerdman [91].

directed to non-regulated firms. Second, the polluting industry in Chapter 3 is not standard, as there exists a positive externality between the two polluting sectors. Interestingly, it can be shown that in these two cases, the property rights payments do not provide correct incentives for efficient allocation.[29]

Overall, the division of the industry into polluting sectors is an important factor. Chávez and Stranlund [7] study the determination of market-based policy between exogenously given polluting industries under uncertainty. Specifically, they allow the unit price of emissions to be different in the two industries. They find a strictly positive margin between the rates, which can be chiefly explained by their assumption about the nature of the uncertainty. Instead of additive type of uncertainty, they assume that the slopes of the marginal benefits are uncertain. Chávez and Stranlund further assume a fixed industry size, so the subsidy payments are not an issue.

The decision to enter to a regulated market is another discrete choice. However, once in the market, the incumbents and new entrants may be treated differently. It is a rather plausible course of action that incumbent firms enjoy subsidies while new entrants do not.[30] Thinking politically, this follows as the incumbent firms are represented in the negotiations while the new firms are not. Differences in treatment are not a problem in our framework, as we can treat the incumbent and new entrants as two separate sectors with different types of subsidization. However, the presence of corner solutions (Goodkind & Wiggins [20]; Nikula [53]) may complicate the policy. The solution is in the corner, if under some realizations of uncertainty, the socially optimal number of new firms equals zero.[31] Goodkind and Wiggins [20] study prices versus quantities under the presence of the corner, while Nikula [53] finds in a two-sector model that the unit price of emissions should be different between the sectors. In this thesis, we assume that uncertainty is small in the sense that

[29] In voluntary participation, the trouble with the property rights payments is that they must be paid to every non-affected firm, whether participating or not. The question is then, why bother to participate if a unit gets a higher payment without participating. On the contrary, the subsidy approach specifically insists that a payment is conditional on the participation, so subsidies have the potential to induce participation. With knowledge spillovers, the trouble with the property rights payments is that they affect the firms' choices exactly the same way as a zero subsidy policy does. We will learn in Chapter 3 that a zero subsidy policy does not represents an efficient policy rule.

[30] New entrants in the Acid Rain Program do not receive allowances for free (Ellerman [15]). However, according to Åhman *et al.* [93], the policies among the member states in the EU differ. The common factor in the policies is that some amount of allowances will be available to new entrants at no cost.

[31] Naturally, corner solutions may emerge with other discrete choices. For instance, under some realizations of the uncertainty, the socially optimal use of green technology may well equal zero.

corner solutions are absent. In practice, the number of active units in the sectors strictly exceeds one under every outcome of the uncertainty.

Recall that the number of technologies in a sector is either one or two. If the number is two, we model the transition from brown towards greener technology. Requate [61] discusses the link between environmental policy instruments and advanced environmental technology. Among other things, he draws a distinction between the adoption, diffusion, and innovation of technology. The term "adoption" refers to investment decisions where firms decide whether to install new existing technology. Specifically, the rate of diffusion is the percentage of firms that adopt the new technology. Models of innovation, in turn, consider the fact that someone must participate in costly R&D so that the adoption can happen in the first place. However, as Requate [61] discusses, the distinction between adoption and innovation is not always clear. For example, in a model of zero R&D, one may argue (like Requate) that the presence of spillovers turns the model toward innovation.

D'Amato and Dijkstra [10], Krysiak [34], and Mendelsohn [42] study environmental investment in a Weitzman [84] framework. Out of these, the model of D'Amato and Dijkstra [10] can be considered as a pure model of adoption. We remark that we deliberately keep our basic structure of adoption extremely simple so that we can model phenomena, such as inefficient substitution, voluntary participation, and spillover effects, in a meaningful way. As the above models include many important details about investments, the instrument choices reflect these details.[32]

Our framework extends to issues beyond the scope of industrial production and regulation in the production markets. For example, one may analyze regulation in the car markets, where both the number of cars and the distribution between clean and polluting cars are simultaneously determined. In this market, a purchase of a green car or the use of public transportation (or both) are being subsidized. Even though it is not a standard idea that car owners are regulated by licenses whose price varies, it is certainly worth investigating.[33] More generally, the framework of the thesis is not only suitable to study discrete investment choices among firms, but also suitable for the study of various household investments (i.e., household technology adoptions) under regulation.

[32]For further references, see Requate [60], Montero [51], and Wirl [90].

[33]One example is the Certificate of Entitlement system in Singapore (Land Transport Authority [35]).

0.3.3 Efficiency Ruined

Meunier [43] reviews the literature of instrument choice where the environmental agency faces more than one restriction.[34] The bulk of the models we analyze in the thesis fall into this category: In Chapters 1 and 2, there is a continuum of subsidy policies that will yield an inefficient allocation of emissions. In Chapter 3, the presence of positive externality calls for subsidies, but it may happen that the regulatory agency is restricted to handling only the negative externality.

Quiron [59] (like Shöb [69]) incorporates public funds raising into his model of instrument choice. Quiron [59] applies a general equilibrium model, where the public budget constraint is explicitly controlled for. The constraint generates a price called a shadow value of public funds that should be part of the environmental regulation. Quiron places a further constraint as he studies non-revenue-raising regulatory instruments. In this regime, the environmental agency is not able to resist the industry pressure and it ends up delivering freely the initial allocations of permits. This happens even when the agency operates under the revenue-raising constraint. The objective of Quirion is more or less the same as ours in Chapter 1, namely, to compare the price and quantity instruments that yield identical expected payment and emission profiles. Our approach omits the public budget constraint. Instead, in our partial approach, inefficient interplay between polluting sectors plays a central role.

Montero [49] studies instrument choice under incomplete enforcement. The environmental agency cannot implement perfect participation, where every regulated unit would report its compliance status honestly. Rather, the agency can sustain only random monitoring, where some firms find it profitable to understate their emission levels. This work is significant from our point of view as it studies instrument choice in a discrete framework. Even though the enforcement is complete in the thesis, the analysis in Montero and our analyses in Chapters 1 and 2 shares many topics and outcomes.[35] In particular, the regulation in Montero's model effectively induces the units in a polluting industry to divide into two separate sectors that together end up

[34] Phaneuf and Requate [56], for example, call the original Weitzman analysis a second-best analysis. The reason for this is the regulator's inability to implement the *ex-post* optimum. In this respect, the restrictions we study in the thesis do not turn the analysis into a second best but merely adds the number of constraints into the model.

[35] Strandlund and Dhanda [78] and Stranlund, Chavez, and Villena [79] examine the implementation of a market-based instrument under inefficient enforcement and continuous choices.

polluting inefficiently. Regarding the volume effect, it evolves as emissions fluctuate under quantity implementation. As some firms in Montero [49] are cheating, the realized aggregate level of emissions will fluctuate away from the steady level of complete regulation.

Recently, Meunier [43] considers instrument choice in a setting where participation can again be interpreted as imperfect. In this setting, unregulated firms pollute alongside regulated firms. The approach is different in our version of imperfect participation in Chapter 2 primarily due to our specific focus on voluntary participation. First, we study implementations where the market will became "less imperfect". In Meunier, the unregulated firms remain outside the regulation all of the time. As a matter of fact, the study of voluntary participation in the Meunier [43] framework is extremely difficult. Second, in the specification of the model, Meunier assumes non-zero cross products in the benefits and damages of emissions, and these cross products create the central effects in the comparative advantage (see also Stavins [73]). Although possible in our framework, we ignore cross products in Chapters 1 and 2. Our effects arise solely from the use of subsidies that enables voluntary participation.[36]

How does our model of voluntary participation compare to other relevant studies? Specifically, how does it compare to the model of Montero [47], which is a pioneering work in this field?[37] The major challenges in Montero's policy implementation are the excessive allocation and the limited transfers. Excessive allocation accumulates as voluntary provision provides permit allocations greater than the counterfactual emissions. This means that the excessive allocation will cover reductions that would have occurred in the absence of the voluntary provision.[38] The limited transfers, in turn, restrict the policy choices of the environmental agency. In practice, the first-best solution is not feasible, as the required allocation of permits is not politically feasible. Note that the permit allocations are akin to the thresholds

[36]Meunier [43] motivates his study in terms of carbon leakages. The term "carbon leakage" refers to global CO_2 policy-making: "*Carbon leakage is defined as the increase in CO_2 emissions outside the countries taking domestic mitigation action divided by the reduction in the emissions of these countries*" (IPCC [24]). We illustrate in Chapter 2 (in a model of imperfect participation without voluntary participation) that our basic framework includes no leakages from affected sector towards non-affected sector. Meunier [43] reviews the literature of carbon leakage for an interested reader.

[37]Bento, Kanbur and Leard [5] provide a recent literature review of opt-in environmental programs. Their review reflects general interest in the CO_2 markets for carbon offsets. To our knowledge, there is not much discussion of an opt-in carbon tax program, not to mention that it is compared to an opt-in carbon offset market.

[38]Bento *et al.* [5] talk about the problems of non-additionality and cap integrity.

in our study.

Our study shares some similarity with Montero [47] but ends up emphasizing different implementation issues. That is, our work is a study of the instrument choice under uncertainty, while Montero studies the design of augmented permit markets. In Montero, the agency takes the threshold of the affected sector (l_A) as given and seeks the second-best level of opt-in firms (l_{NA}). In our approach both thresholds l_A and l_{NA} are taken as given and the choice of the instrument becomes the major task. However, limited transfers cause inefficiency in Montero in the same way inefficient subsidization does in our model. In both works, there exists an ideal means for the efficient implementation of the voluntary provision, but the agency cannot always use those means.[39] Note further that if we replicate Montero's model here (with varying capacities in production), our instrument choice becomes most likely intractable. Consequently, we cannot track the excessive allocation as Montero does. We also emphasize that the volume effect in our framework does not correspond to excessive allocation. Volume effect is entirely an issue in instrument choice. It affects the choice between the price and the quantity instruments but does not affect the design of a single instrument.

Meunier [43] further suggests that the cross products in the aggregate benefit function can be applied in the study of knowledge spillovers. Actually, in Chapter 3, we will apply a certain type of cross product. However, again, our analysis has a different focus than that of Meunier. The important difference is that we concentrate on perfect, rather than imperfect, participation. This means that both sectors remain as affected sectors throughout Chapter 3. Furthermore, the subsidization issue is central to us but it is totally absent in Meunier. Note also the difference between restricted instrument choices in these different approaches. In Meunier [43], the constraint is imperfect participation, while in Chapter 3 of the thesis, the regulatory agency is constrained to ignore the subsidization of positive externalities in production.

[39] Efficiency requires that participation in voluntary program is perfect in Montero [47]. In our model, instead, implementation can be efficient, even though participation is not perfect. Furthermore, participation can be perfect in our model, but the implementation is inefficient.

0.3.4 Efficiency Developed

In addition to the restricted instrument choice, there is another type of choice problem present in the thesis. We say that the environmental agency develops efficiency. More specifically, it develops efficiency by subsidization. This set-up arises in Chapter 3, where multiple externalities are regulated under perfect participation. A central ingredient is that the agency is not restricted, so it is free to pursue efficiency (and even optimality) *ex-ante*.[40] Surprisingly, the implemented policy is unable to retain efficiency. Thus, efficiency is developed and being ruined in Chapter 3. In summary, we study inefficient emission allocations in every chapter of the thesis.

We apply rule $S(s)$ (Equation (6)) in this particular context. The rule depends on the unit price of emissions. We show three points about this choice in Chapter 3. First, the specific linear form of $S(s)$ in Equation (6) is a rule that develops efficiency. Second, in principle, the mere traditional payment rule sl (in Equation (1)) can be adjusted to do the same. Third, the agency can implement only the *ex-ante* efficiency, not the *ex-post* efficiency by the linear payment scheme we suggest. Recall the second-best nature of the Weitzman analysis: The agency is restricted to use only simple instruments; it cannot reset the policy by new information or it cannot publish complicated stage-contingent contracts. However, these means are especially needed in Chapter 3 as the efficiency rule is a stochastic rule. Moreover, the assumed positive spillover causes this phenomenon.

Shinkuma and Sugeta [68] (S&S) is an earlier study of instrument choice under optimal subsidization and multiple externalities. They study instrument choice in the long run when the entry itself creates a positive externality. Importantly, firms face uncertainty at decision time, while positive externality arises from these uncertainties. The model in Chapter 3 shares features with S&S, including that both positive and negative externalities exist in the regulation and that positive externalities call for subsidization. However, the approaches differ in many important ways.

In S&S, the polluting industry consists of *ex-ante* identical firms. They become heterogeneous only after they enter the market and learn their productivity parameter. Our model assumes division to separate polluting industries. Second, the decision-making is sequential in S&S, while in our approach it is not. The sequential decision-making means that a polluting firm first decides to enter, learns its produc-

[40] Terms "*ex-ante*" and "*ex-post*" refer to the moment when uncertainty reveals itself.

tivity, and then chooses the level of production. Specifically, in S&S, not only the environmental agency but also regulated firms face uncertainty and because of that, the assumed entry costs play an integral role in the results. Third, and most importantly, even though both the size of the firms' activities and the number of firms are competitively determined in S&S, the firms' emissions levels are determined *ex-post*, while the number of firms is determined *ex-ante*. Consequently, the number of firms does not react to the outcomes of uncertainty. This is in strict contrast to our analysis, where the number of firms is the sole fluctuating variable. Furthermore, in S&S, uncertainty creates the positive externality, while in our framework (in Chapter 3), it exists in the absence of uncertainty as well. Overall, given these differences, we are unable to track the various effects from S&S that we track from our model in Chapter 3.

0.4 Results

0.4.1 Graphical Presentation of Comparative Advantage

0.4.1.1 Efficient Implementation

The conventional result of instrument choice (Weitzman[84]) says that, even though different instruments yield identical expected prices and quantities, the instruments will most likely differ in terms of expected welfare that they create. The same principle holds in this thesis as well. Next, we review how the instruments differ in terms of expected welfare.

In the original study of instrument choice (Weitzman [84]), the difference between expected welfares, presented as

$$\triangle = (\textit{expected welfare by taxes}) \text{ - } (\textit{expected welfare by tradable permits}),$$

boils down to a simple formula. The choice between instruments is determined by the comparative advantage

$$\triangle = \frac{Var(e)}{2}(\gamma - d), \tag{7}$$

where γ is the slope of the marginal benefits of emissions, d is the slope of the marginal damages of emissions, and $Var(e)$ is the variance of emissions in the price regime.

We have a few comments about the measure. First, the marginal curves are linear. Weitzman utilizes linear approximations, but literature typically assumes the linearity from the outset (Adar & Griffin [1]). Second, the uncertainty itself, as incorporated in $Var(e)$, affects the size of $\triangle$ but does not determine the sign of it. Third, Weitzman assumes uncertainty in both the marginal benefits and in the marginal damages of emissions, but the damage uncertainty is totally absent in $\triangle$. However, if the two uncertainties are correlated, the comparative advantage must be modified to account for the size of the correlation (Stavins [73]).

The instrument choice is determined by the difference $\gamma - d$. A positive value implies that the agency should fix the price in the regulation. In other words, the agency should choose environmental tax as the policy instrument. If the value of $\gamma - d$ is negative, the agency should choose the fixed quantity (tradable permits). For an interpretation, take the case of extremely flat marginal damages ($d \approx 0$). Accordingly, the regulator prefers taxes to the quota. By fixing the opportunity unit cost for polluting, the agency allows the level of pollution to be privately determined. Because of $d \approx 0$, the *ex-post* change in damages is only modest compared to the flexibility that the tax provides on the benefits side. Correspondingly, a very steep marginal damage function favors tradable permits. In this case, the trouble in benefits that a fixed level of pollution induces is only modest in comparison to the potential increase in damages that a fluctuating level of pollution would create. Finally, if $\gamma = d$, the environmental agency is indifferent between prices and quantities.

We provide a graphical illustration for the comparative advantage.[41] The illustration employs welfare triangles that grow up, as the agency is not able to implement *ex-post* efficiency. In Figure 2(a), we have drawn the marginal damage function (md), the expected marginal benefit function (Emb), and one specific outcome of the marginal benefit function (mb_d). The price instrument is a unit tax set at τ, while the use of quantity instrument means fixing the emissions at level L. Note that the (expected) stringencies of these policies are based on rule $\tau = Emb(L) = Emd(L)$. However, as compared to *ex-post* efficiency, the chosen instrument will inevitably produce a certain welfare loss. The black spot denotes the *ex-post* efficiency in the

[41] Baumol and Oates ([4], Chapter 5) provide a very careful graphical interpretation of the comparative advantage.

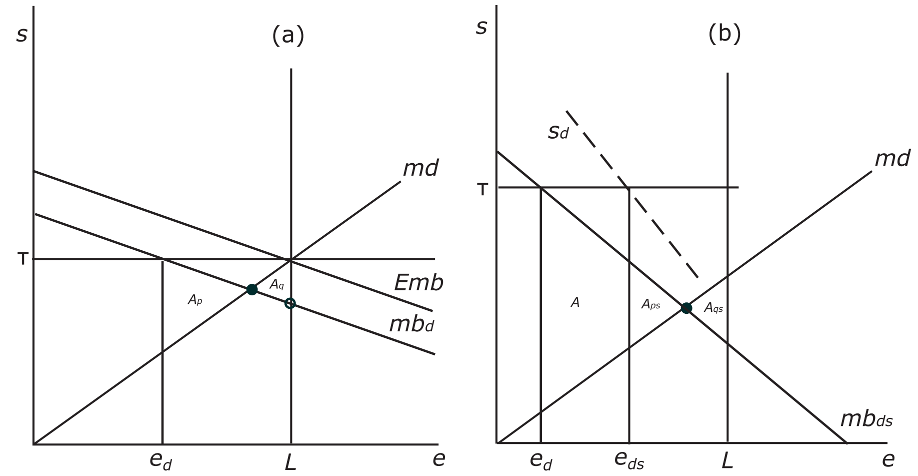

Figure 2 The Instrument Choice under Uncertainty (a); The Subsidized Instrument Choice (b)

figure. It is the price—quantity pair that satisfies condition $mb_d = md$. With taxes instead, the price is fixed to τ, but the *ex-post* quantity response is e_d. With tradable permits, the quantity is fixed to L, but the *ex-post* price response satisfies $p = mb_d(L)$. The small circle in the figure represents this price. Overall, area A_q is a typical welfare loss that the commitment to a quantity instrument produces. Alternatively, area A_p is a typical welfare loss that the commitment to a price instrument produces. Clearly, $A_p > A_q$ in Figure 2(a), so the agency should choose the quantity instruments. Note that we have $d > \gamma$ in Figure 2(a), so that we would have reached the same conclusion by using the rule in Equation (7).

0.4.1.2 Inefficient Implementation

We have drawn Figure 2(b) to illustrate the determination of the subsidized comparative advantage. It explains graphically the principal difference to the more standard approach of Figure 2(a). In doing that, it utilizes the difference between the marginal benefit function and the so called price function.

We repeat first a property that underlies Figure 2(a): the marginal benefit function also operates as a reaction function of the polluting industry. This means that we can read both the price and the quantity responses and the changes in welfare by tracking the movement of the curve mb_d. The reason for this is that the implementation is efficient. We will follow the literature as we label the curve mb_d as a marginal abatement curve. In Figure 2(b), we depict an inefficient implementation. Now, i) the marginal benefit function no longer equals the marginal abatement function and ii) the reaction function no longer equals the marginal benefit function. We label the reaction function as the price function. We derive and explain the price function $s(e;\theta)$ in Section 1.3.4 (see Equation (1.40)).

In Figure 2(b), we do not specify the policy regime but merely assume the presence of inefficient subsidization. In particular, the figure displays the stringencies of the policies by parameters τ and L, but does not explain how these values are determined.[42] Figure 2(b) also includes specific outcomes of the marginal benefit function (mb_{ds}) and the price function (s_d). The inefficiency causes $mb_{ds} \neq s_d$. Consequently, the price and quantity responses are no longer determined by the shifts in mb_{ds} but rather by the shifts in s_d. In terms of Figure 2(b), the quantity response in the tax regime is e_{ds} *not* e_d. Areas A_{qs} and A_{ps} are welfare losses for the quantity and price instruments, respectively.[43]

We have a few brief comments here and save more complete discussions for the main text. First, we have not applied mb_{ds} and s_d only for the sake of presentation. Rather, the dual representation is needed so that the graphical analysis can be performed in the standard quantity-price-plane. Second, we called areas A_p and A_q “typical" in Figure 2(a). By this, we mean that a single pair of welfare loss triangles truthfully reflects the comparative advantage. In Figure 2(b), we can call areas A_{qs} and A_{ps} typical as well. The reason is that the deviation e_{ds} is typically smaller than e_d under inefficiency. Third, the welfare losses A_{qs} and A_{ps} are determined by the same logic as they are in Figure 2(a). The *ex-post* efficiency is denoted by a black dot in Figure 2(b), where it holds that $mb_{ds} = md$. This would have been the policy choice had the agency learned the uncertainty and revised the policy. Moreover, as compared to the efficient implementation, an area denoted by A is missing in Figure 2(b). By graphical reasoning, this area illustrates the price advantage that the inefficient subsidization induces.

We review this topic more closely in the next section. We discuss how the inefficiency alone favors the price instrument, how there are different types of inefficiencies, and how factors other than inefficiency affect instrument choice. We will learn that the outcome in Figure 2(b) is a specific one, as it displays only the pure influence

[42]We will explain the policy choices at length in Section 1.3 of Chapter 1.

[43]To keep the figure simple, we have not marked the price response in the quantity regime. It is determined by the intersection of lines s_d and L.

of inefficiency. In particular, it dismisses the possibility that emissions may fluctuate under quantity implementation as well. In Figure 2(b), the emissions are constant and equal to L under quantity implementation.

0.4.2 Augmented Comparative Advantage

Figures 2(a) and (b) together illustrate how the determination of comparative advantage differs between the efficient and inefficient subsidizations. Next, we explain more formally how the switch from an efficient to an inefficient subsidization changes the comparative advantage. We write the so-called augmented comparative advantage as

$$\triangle = R(\gamma - r_1 d) + r_2, \tag{8}$$

where R, r_1, and r_2 are fixed parameters. Again, γ is the slope of the marginal benefit function and d is the slope of the marginal damage function. The augmented measure in Equation (8) allows us to write every comparative advantage in this work by merely changing the values of R, r_1, and r_2. Specifically, $R = \frac{Var(e)}{2}$, $r_1 = 1$, and $r_2 = 0$ in the original Weitzman presentation. In our models to come, we have $R > 0$, but the signs of the factors r_1 and r_2 will vary between different specifications.

0.4.2.1 Cost Effect and Volume Effect

Throughout the thesis, we use the terms "cost effect" and "volume effect" to explain the influence of inefficient subsidization on the comparative advantage. We can use these terms to explain the values of r_1 and r_2 in Equation (8) as well. The cost effect gives the pure effect of inefficient subsidization in the sense that emissions are kept fixed in the calculation. Actually, Figure 2(b) above is all about the cost effect, as the policy keeps the amount of emissions fixed at the level L. The volume effect takes into account the fact that the quantity implementation does not always fix the emission quota. The volume effect relates fundamentally to the fluctuating number of firms in the implementations that apply emission thresholds. In the thesis, the number of firms fluctuates within the sectors and within the technologies. The commitment target of the environmental agency in part determines the existence of the volume effect. Specifically, the agency may or may not let the volume effect happen.

To explain the volume effect, it generally holds that

$$\lambda_0 l_0 + \lambda_1 l_1 + l = L,$$

where l is the number of auctioned permits, L is the number of aggregate permits, and $\lambda_0 l_0$ and $\lambda_1 l_1$ are the number of subsidized permits in sectors zero and one, respectively. Furthermore, if the policy regulates every polluter in the industry, then L equals the aggregate level of emissions. As far as the number of firms (λ_0 and λ_1) will fluctuate, the realized level of emissions strongly depends on the fact of whether L or l is fixed in the policy. In particular, by fixing L, the agency allows l to adapt to keep the emissions at the predetermined level. Alternatively, by fixing l, the agency will deliberately let the quota (and the emissions) fluctuate. The volume effect is present in the second design and absent in the first one.[44]

In this context, we will apply the terms “sterilized" and “non-sterilized". The permit market outcome is sterilized whenever the agency changes the number of auctioned permits l in response to fluctuations in the market. The agency practically sterilizes influences that the changing number of polluting units create on the total permit quota. Instead, in a non-sterilized outcome, no such amendments occur.[45] If we were to follow the original Weitzman [84] description literally, then we should study only non-sterilized permit implementations. In this interpretation, the authority is reluctant to reset its policy, so sterilization is not applied. However, we can think of the policy in terms of commitments as well. In Weitzman [84], the specific question of commitment does not arise. That is, whether the agency commits to the number of auctioned permits or to the number of total permits, the level of emissions remains fixed. Here, the commitment to the fixed number of auctioned permits means that the agency is not committed to the fixed levels of emissions. We think that we cannot justify any commitment at the outset. Rather, institutional

[44]The hybrid instrument by Roberts and Spence [62] implements a special kind of volume effect. The instrument itself is a mixed implementation, whereby a non-linear tax scheme complements the tradable permit markets. The scheme sets a floor and a ceiling to a permit price. Consequently, it allows strictly lower and higher aggregate emissions than the fixed quota does. Recent contributions in this area have been made by Krysiak and Oberauner [33] and Mandell [40].

[45]There are earlier studies of instrument choice under uncertainty that include comparisons between different quantity instruments. For example, Yates and Cronshaw [92] study whether permit trading between different compliance periods should be allowed or not. Montero [48] in turn asks whether different pollution markets should be integrated or not.

issues will determine it or it may be a choice variable. That is, the environmental agency may have the power to pick the commitment that will yield the highest social welfare.[46]

0.4.2.2 Efficiency Ruined

In the models of restricted instrument choice (Chapters 1 and 2), inefficiency arises from the fact that the view of the regulatory agency does not determine the subsidization policy. We show that values $r_1 \lesseqgtr 0$ and $r_2 = 0$ are plausible in this particular context. As far as $r_2 = 0$, factor r_1 includes both the cost effect and the volume effect. If $r_1 < 1$, we say that the cost effect dominates, and if $r_1 > 1$, the volume effect dominates. Specifically, if $r_1 > 1$, we say that the relative position of the tax instrument has increased. We will further show that the volume effect requires inefficient subsidization to work. In other words, the absence of the cost effect implies the absence of the volume effect. However, the reverse is not true, as the cost effect may well arise even if the volume effect remains absent. Finally, efficient implementation means that the cost effect is absent and that $r_1 = 1$ and $r_2 = 0$.

We discussed earlier the restricted instrument choices of Quirion [59] and Montero [49]. In terms of the comparative advantage in Equation (8), there is certain similarity between our studies of restricted instrument choice and these two studies. In particular, in every three cases, $r_2 = 0$.

If we denote by SH the shadow value of the public funds, then $r_1 = \frac{1}{SH}$ in Quirion [59]. As far as $SH > 1$, then $r_1 < 0$, so the presence of public budget constraint improves the relative position of taxes. A special feature in Quirion is that $r_1 = \frac{1}{SH}$ whether revenue-raising or non-revenue-raising instruments are applied. A revenue-raising instrument refers to wholly auctioned permits. On the contrary, we will show in Chapter 1 that a revenue-raising instrument is always efficient, so $r_1 = 1$. These differences arise from the difference between partial (our study) and general equilibrium analysis (Quirion's study). Specifically, if lump-sum taxes are available, then $SH = 1$ and the difference between revenue-raising and non-revenue-raising instruments disappears in both studies. In our partial analysis, the value of

[46] One may wonder how the agency implements the sterilized systems. In Chapter 1, we propose and present a system called a discount coupon system, in which every freely allocated permit represents a coupon that can be exchanged for the actual license in the permits auctions. Note also that the non-sterilized system does not require any coupons. The endowment of permits can be auctioned off in a straightforward manner.

r_1 is shown to be a function of the subsidy policy (l_0, l_1) while r_1 is exogenously determined in Quirion [59]. Furthermore, Quirion's general equilibrium analysis precludes the presence of volume effects.

Montero's study [49] shares the partial equilibrium nature with our study. If we denote by π the probability of being randomly monitored, then $r_1 = 2 - \pi$ in Montero [49]. As $0 > \pi > 1$, then $r_1 > 0$, so the incomplete enforcement improves the relative position of tradable permits. Using the terminology of our thesis, both cost and volume effects are in place but the volume effect is seen to invariably dominate in Montero [49]. We say that the cost effect is in place as emission allocation between sectors is inefficient in Montero. The volume effect exists because the emissions are not quoted with probability one.

In our models to come, we will track both cost and volume effects but, in general, neither effect will dominate. Rather, as the total effect r_1 is a function of the subsidy policy (l_0, l_1), the various thresholds will pull r_1 to values that are both greater and smaller than one.

0.4.2.3 Efficiency Developed

The above examples considered restricted instrument choice. We also study another type of instrument choice where efficiency is developed rather than ruined. Our specific question concerns the instrument choice under multiple externalities. In analyzing this issue, we note first the fundamental difference from the previous analysis. In Chapters 1 and 2, if the allocation rule is efficient, it is invariably deterministic and the instrument choice follows the standard Weitzman [84] analysis. In Chapter 3, the allocation is efficient, which can make us think that the instrument choice always follows Weitzman. However, this is not the case. This is mainly because the spillover effect turns the efficiency rule stochastic.

We review two designs in Chapter 3. The first policy is an optimal policy that internalizes the knowledge spillover by setting a proper subsidy in the externality-generating sector. The second policy is suboptimal as the social welfare is maximized under the constraint that the externality-generating sector cannot be subsidized. We consider briefly the first policy design here. Regarding the comparative advantage in Equation (8), we show that $r_1 > 1$ and $r_2 > 0$. Furthermore, we call factor r_1 the slope effect while r_2 is the cost effect. The slope effect is seen to favor quantities while the cost effect favors prices.

The parameter r_2 represents cost effect. This follows as the quota in Chapter 3 remains fixed by assumption. The slope effect belongs to a group that we denote as base effects. In addition to cost and volume effects, we identify base effects as separate types of effects in instrument choice. A base effect in practice allows us to switch from one efficient representation to another.[47] It covers effects on the benefit side that are missing in the traditional Weitzman analysis and that are not the result of inefficiency. In the current case, the slope effect arises as the assumed knowledge spillover reduces the slope of the marginal benefit function. As we write our augmented comparative advantage in terms of γ (the traditional slope parameter without the spillover effect), $r_1 > 1$ records the specific effect of knowledge spillover. By the basic principles of instrument choice, as the marginal benefit curve becomes less steep, more weight will be given to the (constant) slope of marginal damage d. In other words, as the relative importance of pollution damages increases, the relative importance of quantity control increases.

We can further illustrate our approach with a reference to the study of Mendelsohn [42]. The Mendelsohn study can be regarded as an early milestone in this field. It studies how endogenous technical change affects the instrument choice between a price and quantity instrument. In terms of our augmented comparative advantage, Mendelsohn finds that $r_1 > 1$ and $r_2 = 0$. In his model, R&D in clean technology makes the abatement cost curve less steep, that is, less responsive to the changes in price. This is reflected in the value of $r_1 > 1$, which means that R&D in clean technology improves the relative position of the quantity instrument. Consequently, in both studies, r_1 reflects the influence in the instrument choice that the transition towards green technology generates. In Mendelsohn, the change is from zero to positive R&D, while in our study, the change is from zero to positive spillovers. Second, factor r_2 equals zero in Mendelsohn [42]. This implies that inefficient subsidization is not an issue in his study.

0.5 The Outline

Chapter 1 presents the fundamental model of instrument choice under inefficient subsidization. There is a polluting industry with two polluting sectors. The envi-

[47] Another base effect is the scope effect that we discuss briefly in the next section and more broadly in Chapter 2.

ronmental agency does not have total control over the policy. In particular, there are cases in which the agency strongly disagrees with the proposed distribution of subsidies. In general, the goal of the analysis is not to focus on the causes but rather to look at the consequences. We explore the properties of a feasible policy in a framework in which the agency sees the subsidies as a constraint on its policy. As in Weitzman [84], the main policy object is the instrument choice in the event of uncertainty.

The analysis in Chapter 1 isolates two effects: the cost effect and volume effect. The cost effect depicts the pure influence that the inefficiency induces on permit trading. The analysis shows that this effect invariably favors stable prices, so the tax system with a fixed price has an unambiguous advantage. The volume effect takes into account the fact that the quantity instrument does not fix the emissions. In fact, in implementing the quantity instrument, the agency has the power to decide whether to stabilize the emissions. We say that the agency may either commit to the number of total permits or to the number of auctioned permits. In the latter implementation, the volume effect rises, but after being born, it is shown to favor either the price or the quantity instrument. The analysis also reveals that the volume effect requires inefficient subsidization to work. In other words, the efficient subsidization automatically produces a zero volume effect.

Our identification of the volume effect will divide the quantity instrument into two separate instruments, namely into sterilized and non-sterilized permit systems. We say that the agency sterilized the volume effect away. As the price instrument remains an option, it is now possible that the choice is between three instruments. Naturally, the agency will sterilize as long as the comparative advantage between the quantity instruments shows that it is useful. Overall, we will construct and review at length the new type of choice problem between three distinct instruments, both analytically and graphically.

Chapter 2 applies the framework presented in Chapter 1. It studies inefficient subsidization in the implementation of a particular environmental policy. The discussion concerns imperfect participation of the regulation, and, specifically, a voluntary provision that is capable of alleviating problems of imperfectness. Imperfect participation means that not every unit that contributes to the (negative) externality is involved in the regulation. The imperfectness can be seen as another constraint in the regulation. This means that the environmental agency prefers regulation where everyone is involved (perfect participation), but the distributional and political realities block the achievement of this goal (Montero [47]). We show that implementation of voluntary participation requires subsidies, and in accordance with the theme of the thesis, the presence of subsidies immediately raises potential inefficiencies. The use of heavy subsidization does not automatically ruin efficiency, but the requirements for efficiency are specific in nature.

The instrument choice is again exposed to the cost and volume effects. In this particular context with imperfect participation, some specific phenomena will arise. First, we will introduce the concept of scope effect.[48] This effect refers directly to values of γ and r_1 in the augmented comparative advantage (Equation (8)) above. Remember that γ is the slope of the (efficient) marginal benefit function, while r_1 generally includes the combined cost and volume effects. Generally speaking, the scope effect in voluntary participation concerns the choice of γ, or equivalently, the choice of a representative efficient market. We will base parameter γ on the imperfect market chiefly to set a proper point of reference. Then, our results have a natural interpretation that they represent changes *toward* perfect participation, that is to say, represent changes when the market will become "less imperfect". As for the scope effect itself, note that the increased participation will make the benefit curve less steep. Naturally, this requires that the policy successfully attracts cost-effective projects to participate. From the basic principles of instrument choice, more weight will be given to the (constant) slope of marginal damage d as the marginal benefit curve becomes less steep. In the comparative advantage, the value of r_1 increases as the relative importance of pollution damages increases.

In the absence of volume effect, we may call r_1 as "the combined cost-scope-effect." The analysis in Chapter 2 shows that two forces will pull this measure into opposite directions. The inefficient subsidization increases the scope of the regulation but causes a cost effect. Consequently, the value of r_1 decreases. However, the increase in the scope of the regulation (the scope effect) increases the value of r_1, and it may actually increase it so much that eventually $r_1 > 1$.

Another specific phenomenon relates to the nature of the volume effect. In general, the volume effect will disappear if the agency will and is able to stabilize the aggregate emissions. In Chapter 2, an important detail in the policy concerns the coverage of the voluntary provision. In particular, it matters whether the voluntary provision covers the entire non-affected sector or whether it does not. In the first of

[48]Scope effect is another base effect. We discussed the concept of base effect in the previous section.

these cases, a commitment to keep the aggregate quota at a predetermined level stabilizes the emissions. If the entire non-affected sector is not covered, then a fixed quota does not yield fixed emissions. A policy target that deliberately fixes the number of permits but does not fix the level of emissions sounds like a weird policy target. We will ignore this type of implementation in Chapter 2.

In Chapter 3, firm subsidization and instrument choice remain the main subjects of the study, but the perspective changes a bit. Specifically, the focus is on instrument choice in a situation where subsidization is particularly required as part of an efficient environmental policy. We show that the reason for this is the presence of a positive externality—the knowledge spillover. If the policy does not subsidize the externality-generating sector at all, it will not adequately produce the externality. On the other hand, we assume that both sectors in the economy produce the same negative externality. Consequently, if the policy does not ration the aggregate production of it at all, the polluting industry overproduces the externality. The policy is then implemented under multiple externalities.

Overall, the agency will again face the standard Weitzman [84] constraint that it cannot totally revise the policy to reflect the changing regulatory regime. However, as compared to earlier chapters, this constraint has a new meaning in Chapter 3. The knowledge spillover inside the polluting industry is shown to create new needs for policies that cannot be solved under the Weitzman assumption. Traditionally, if the policy design yields efficiency *ex-ante*, it will yield efficiency *ex-post* even though the regulation operates under the Weitzman constraint. Now, this principle no longer holds. Emission allocation will turn inevitably inefficient *ex-post*, and whenever allocation is inefficient, it will affect the choice between prices and quantities. It is further shown that the inefficiency is transformed into pure cost effect, and like in every chapter of the thesis, the cost effect will favor the price instrument, that is, environmental taxation.

In practice, the optimal expected policy in Chapter 3 promotes subsidization but it does not explicitly state the form of subsidization. However, by closer inspection, the expected welfare under uncertainty depends heavily on the chosen instrument. Indeed, this is the same type of case that Weitzman [84] presented almost 50 years ago. Weitzman shows how the optimal policy promotes the pricing of emissions but does not explicitly state whether prices or quantities should be applied. In our case, we argue for a linear subsidy rule that explicitly depends on the unit price of emissions. Thus, in comparing prices and quantities, we compare implementations, where one implementation applies tradable permits with a stochastic subsidy rule while the other applies environmental taxation with a fixed subsidy. We admit that other implementations will emerge (we even discuss them) that will affect the comparative advantage in different fashions. However, we remark that a zero subsidy implementation does not reduce the instrument choice back to the original choice. Rather, it only creates a new type of choice.

We explained above how another base effect, a slope effect, will affect the instrument choice in Chapter 3. The slope effect complements the instrument choice because knowledge spillover will reduce the slope of the marginal benefit function. This will favor the quantity instrument, so the slope effect and the cost effect will eventually pull into different directions.

We will provide a summary of the main results of this thesis in the Conclusions section. We also discuss the meaning of our key assumptions along with possible future research topics motivated by this work. The appendices provide various proofs with somewhat tedious calculations.

1 INEFFICIENT SUBSIDIZATION

1.1 Some Background

In the field of environmental regulation, the study of the choice of policy instruments dates back to 1974 when Martin Weitzman [84] published his influential paper, "Prices vs. Quantities." He shows that not only the goal but also the means are important in environmental regulation. Weitzman postulates that regulatory decisions are most often made in a situation in which both the benefits and damages are uncertain. With respect to policy implementation in this situation, Weitzman promotes simplicity as a viable criterion. In fact, he limits the available administrative instruments, that is, the control modes, to fixed price and fixed quantity. Furthermore, Weitzman ([84], p.482) states that "... *the consequences of an order given in a particular control mode have to be lived with for at least the time until revisions are made*." In practice, this view sets up a certain kind of leader–follower game, in which the regulatory agency (the leader) sets the control mode for a significant length of time while the regulated units (the followers) adapt to the control and to the changes in their business opportunities.

In comparing the two control modes, Weitzman disregards monetary payments between the regulator and the regulated.[1] However, the instruments in his study—pollution tax and pollution quota—yield very different payments. By the very nature of the tax instrument, the magnitude of the emission determines the polluter's payment. Conversely, the emission quota induces a zero environmental payment regardless of the size of the quota. Of course, ignoring the payments can be justified by the fact that the streams do not affect the choices of the parties of regulation. Specifically, they do not necessarily affect the choices of the regulated. Rather, the payment may only reduce the existing producers' surpluses.

[1] The environmental payment is the unit price of emission multiplied by the emitted quantity.

We study instrument choice in a model, where the total payment is an integral factor of the problem. Instead of studying the regulation of a single polluter, we will concentrate on the regulation of numerous polluters. The model includes two polluting sectors that are heterogeneous but that have at least one thing in common: the polluting units within the sectors emit exactly the same type of pollution. In fact, we assume that the two sectors together cover all emissions, so they are said to constitute a polluting industry. The differences between sectors follow from the production of different types of commodities, while the differences within a sector are due to differences in efficiency. An additional difference between the sectors is that the intensities of emission generation differ between them. Furthermore, a sector can be viewed either as an existing or as a prospective sector. In every case, the regulatory agency faces a choice between two market-based instruments: Piqouvian taxes and tradable permits. Most importantly, the instruments trigger payment flows between the regulator and the regulated sectors, and these flows affect the production decisions of the firms. Furthermore, we base our model on the (reasoned) view that the two instruments are *ex-ante* identical in terms of their monetary payments.

To better understand our approach, we draw a distinction between market-based (or economic-incentive) policy instruments and "command-and-control" regulations at the outset (see also Stavins [75]). Market-based instruments are regulations that "*encourage behavior through market signals rather than through explicit directives regarding pollution control levels or methods*" (Stavins [75], p. 9). The use of market-based regulation is particularly powerful when the number of regulated firms is large and the firms have private information that is not costlessly available to the environmental agency. The basic argument is then similar to that for traditional commodities: The market solution is the most efficient way to distribute an allocation of goods. Thus, market-based emission allocation between the polluters is the most efficient way to distribute any amount of total emissions.

In practice, market-based instruments create a price for emissions. Ideally, this price will guide polluting units toward efficient allocation. However, based on the nature of trading, market-based instruments also induce payment flows between the regulator and the regulated. The question is whether the payment flows significantly affect the allocation process. We construct a simple model of discrete choices to study this question. The advantage of our model is its ability to focus on elaborate questions. Specifically, we are able to focus on the number of polluting units in the industry.

The term “subsidy" is a central one. The regulated firm enjoys a subsidy if it does not pay for each pollution unit that it generates and the subsidy is paid only to firms that produce output (Pezzey [54]). With taxes, the subsidy begins at a certain threshold, so that the unit tax is paid only for the emissions that will exceed the threshold. Conversely, with tradable permits, a subsidy is a free allocation of permits. In every case, if the subsidy is zero, the environmental payment flow is maximal. Pezzey [54] summarizes the discussion about the long-run properties of the different instruments, noting that, in general, “the polluter pays” principle provides the correct long-run incentives.

In our analysis, we concentrate on inefficient subsidies that are exogenous to the environmental agency. We incorporate this view into social welfare maximization. In particular, we will study the influence of the different subsidies on the instrument choice. In our interpretation, the decision-making unit (the agency) is a public institution, where partial optimization is an integral part of the decision-making process. Within this type of organization, the agencies choose some parts of the policies, while they take some parts of them as given.[2] In our case, the policy as a whole includes decisions about the proper level of subsidies, the strictness of the policy, and the instrument applied in the regulation. We then assume the first part as given and allow the second and third decisions to be endogenous. In this respect, the status of the regulatory agency is quite special in the original Weitzman study. It is best described by the term “omnipotent benevolence." Clearly, this type of agency does not face the challenges of partial optimization.[3]

We label our regulator (or the regulatory agency) as restricted. In the literature on instrument choice, the idea of a restricted regulator is not entirely new. For example, Montero [49] applies it in his Weitzman framework when studying instrument choice under incomplete enforcement. Even though he does not call the regulator restricted, the similarity to our regulator is obvious. In his model, the regulatory agency can determine the strictness of the policy and the instrument used in the implementation, but it takes the details of the enforcement policy as given. Specifically, the maximum fine remains beyond the control of the regulator. The maximum fine

[2]The enforcement of the policies based on hierarchical decision-making is studied, for example, in Jones and Scotchmer [26].

[3]Miyamoto [45] models the political process explicitly in his study of instrument choice. In particular, he focuses on the lobbying activity of the polluting firms.

is assumed to be low enough that the enforcement is incomplete. Our agency decides on the strictness of the policy and the instrument in the implementation but takes the details of the distributional issues as given.

In both studies, inefficient allocations of emissions between the polluters reflect the lost omnipotence of the agency. Furthermore, in both studies, inefficiency affects the decision rules between the instruments so that the original Weitzman rule no longer applies. In Montero [49], a parameter in the enforcement policy affects the decision rule. In our study, the rule depends on sector-specific subsidy thresholds.

We show that the original Weitzman analysis remains valid only when the subsidization is efficient. For example, we show that the celebrated polluter pays principle represents a case in which both efficiency and the original Weitzman rule hold. However, we do not take for granted that this principle (justified on efficiency grounds) is typically implemented. Our question then concerns the content of Weitzman's decision rule in numerous cases where the efficiency criteria do not hold. In this sense, our study complements the literature on instrument choice.

Our analysis finds that inefficient subsidization has two novel effects on the instrument choice: the cost effect and volume effect. The cost effect depicts the influence that inefficiency induces on permit trading. This effect invariably favors stable prices, so the tax system with a fixed price has an unambiguous advantage. The volume effect takes into account the fact that quantity regulation no longer fixes the emissions. Actually, in implementing the quantity instrument, the agency has to be more specific about the emission quota. In our particular context, the agency may either commit to the number of total permits or to the number of auctioned permits. In the latter implementation, we show how the volume effect arises, but it may favor either the price or the quantity instrument. We also show that the volume effect requires inefficient subsidization to work. In other words, efficient subsidization automatically produces a zero volume effect.

1.2 The Model

1.2.1 Polluting Industry

The polluting industry consists of two distinct sectors labeled 0 and 1. The benefit for unit λ after producing one commodity in sector i becomes

$$B_i(\lambda) = b_i + \theta - c_i \lambda, \tag{1.1}$$

where b_i and c_i are positive constants and $i = 0, 1$. The variable θ is a random variable with $E(\theta) = 0$ and $Var(\theta) = \sigma^2 > 0$. The net benefit is

$$\Pi_i(\lambda) = B_i(\lambda) - s(\alpha_i - l_i), \tag{1.2}$$

where α_i is the level of emissions that the production of one commodity in sector i generates and $s(\alpha_i - l_i)$ is the monetary payment. In the payment function, l_i is a sector-specific threshold level and s is a unit price of emissions common to both sectors. The monetary payment allows the use of both tradable permits and environmental taxes. We let $s = p, \tau$ with permits and taxes, respectively. In case of permits, threshold l_i is the initial allocation of permits to a production unit, while with taxes, l_i is the tax-free level of emissions. In both cases, $s l_i$ is a subsidy payment, so it is paid only to an active unit.

We denote the number of active units in sector i by λ_i. It holds that

$$\Pi_i(\lambda_i) = 0, \tag{1.3}$$

so

$$\lambda_i(\theta) = \frac{b_i + \theta - s(\alpha_i - l_i)}{c_i}, \tag{1.4}$$

where $i = 0, 1$. We denote the level of emissions in sector i by e_i, then

$$e_i = \int_0^{\lambda_i} \alpha_i d\lambda = \alpha_i \lambda_i, \tag{1.5}$$

where $i = 0, 1$. We assume that the pollution is homogenous in nature. Then, the total level of pollution as

$$e = e_1 + e_2 = \lambda_0 \alpha_0 + \lambda_1 \alpha_1 \tag{1.6}$$

will be of interest.

We will derive our results by using the condition $\alpha_i - l_i \geq 0$, $i = 0, 1$. These conditions can be seen as setting upper limits for the subsidization policy. By Equation (1.4), the policy will reduce polluting units (and the pollution) in both sectors. However, these conditions also imply that both sectors have permit deficits at the equilibrium, so they have positive demands for permits in permit auctions. We note that this type of market structure is not the only plausible one. In particular, some firms may modify their processes in such a way that they end up having permit surpluses at the regulatory equilibrium. They may modify their product lines by switching to less polluting inputs or they may install end-of-pipe purification technologies. We will discuss this issue towards the end of this chapter. Until then, our assumption provides a simple framework for building a basic intuition.

1.2.2 The Regulation

The regulation implements the environmental policy with a market-based instrument. In practice, the environmental agency chooses between tax and permit instruments. Both instruments operate by setting a unit price s on emissions. The tax instrument sets directly $s = \tau$. The permit policy, instead, fixes a number of permits, and price $s = p$ is determined in the permit markets. In both cases, the regulation is implemented under an additional assumption that the units are subsidized by sector specific emission thresholds.

Let us discuss the permit implementation in more detail. We write a general relation between aggregate and auctioned permits as

$$L = \lambda_0 l_0 + \lambda_1 l_1 + l, \tag{1.7}$$

where L is the number of aggregate permits, l is the number of auctioned permits, and λ_0 and λ_1 are the number of units in sectors 0 and 1, respectively. The formula depicts a mixed permit handout, where a part of the total allocation $(\lambda_0 l_0 + \lambda_1 l_1)$ is given out for free while a part of the allocation (l) is fully charged for during permit auctions.

We make a distinction between two commitments. The environmental agency may commit either to the number of aggregate permits or to the number of auctioned permits. The question of commitment becomes particularly important when we allow for the presence of uncertainty. In dealing with uncertainty, we assume a particular order of moves between the agency and the regulated industry. This assumption is due to Weitzman [84], and it sets up a certain kind of leader–follower game. We assume that the agency set the regulatory parameters before everyone learns of the uncertainty, and after that, the agency is unable to re-optimize. The chosen regulatory parameter is either the number of permits or the tax rate. Firms make all their choices only after the regulation is fixed and after uncertainty presents itself.

If the policy fixes the number of the auctioned permits at level l, the number of total permits becomes

$$L(\theta) = \lambda_0(\theta)l_0 + \lambda_1(\theta)l_1 + l, \tag{1.8}$$

where the variables $\lambda_0(\theta)$ and $\lambda_1(\theta)$ are defined in Equations (1.4). Since $\lambda_0(\theta)$ and $\lambda_1(\theta)$ are not constants, the total number of permits becomes variable. Alternatively, the policy may fix the total number of permits at level L. Then the number of auctioned permits is

$$l(\theta) = L - (\lambda_0(\theta)l_0 + \lambda_1(\theta)l_1), \tag{1.9}$$

so that this time the number of permits to be auctioned off becomes variable. We feel that there is no good *a priori* reason to favor one definition over the other, so we will handle both cases. However, if the total number of permits is fixed, Equation (1.9) shows how the agency must *sterilize* the consequences that the entry-exit decisions create, and the solution is, therefore, a *sterilized* solution. Note also that there is no intrinsic need to re-define the tax policy because the regulatory parameter of interest (the tax rate) is truly fixed, and, as such, is entirely independent of the tax exemptions.

Let us solve the equilibrium prices under the various commitments. In the sterilized system, the (fixed) supply of permits equals L, the number of aggregate permits. The market equilibrium equates supply and demand of permits, so it satisfies

$$\lambda_0(\theta)\alpha_0 + \lambda_1(\theta)\alpha_1 = L.$$

We insert the variables $\lambda_0(\theta)$ and $\lambda_1(\theta)$ (Equation (1.4)) into the equilibrium condition, so

$$\frac{b_0+\theta-p^L(\alpha_0-l_0)}{c_1}\alpha_0+\frac{b_1+\theta-p^L(\alpha_1-l_1)}{c_1}\alpha_1=L. \tag{1.10}$$

After arranging terms, the price becomes

$$p^L=\gamma^L\left(\frac{b_0\alpha_0}{c_0}+\frac{b_1\alpha_1}{c_1}-L\right)+\gamma^L\left(\frac{\alpha_0}{c_0}\theta+\frac{\alpha_1}{c_1}\theta\right), \tag{1.11}$$

where

$$\gamma^L=\frac{c_0c_1}{c_1(\alpha_0-l_0)\alpha_0+c_0(\alpha_1-l_1)\alpha_1}. \tag{1.12}$$

We further write

$$p^L(\theta)=\overline{p}^L+\gamma^L\left(\frac{\alpha_0}{c_0}\theta+\frac{\alpha_1}{c_1}\theta\right), \tag{1.13}$$

where

$$\overline{p}^L=E\left[p^L(\theta)\right]=\gamma^L\left(\frac{b_0\alpha_0}{c_0}+\frac{b_1\alpha_1}{c_1}-L\right). \tag{1.14}$$

In the non-sterilized system, the (fixed) supply of permits equals l, the number of auctioned permits. The market equilibrium satisfies

$$\lambda_0(\theta)(\alpha_0-l_0)+\lambda_1(\theta)(\alpha_1-l_1)=l. \tag{1.15}$$

After inserting the variables $\lambda_0(\theta)$ and $\lambda_1(\theta)$ into the equilibrium condition, we can write the equilibrium price as

$$p^l=\gamma^l\left(\frac{b_0(\alpha_0-l_0)}{c_0}+\frac{b_1(\alpha_1-l_1)}{c_1}-l\right)+\gamma^l\left(\frac{\theta(\alpha_0-l_0)}{c_0}+\frac{\theta(\alpha_1-l_1)}{c_1}\right), \tag{1.16}$$

where

$$\gamma^l=\frac{c_0c_1}{c_1(\alpha_0-l_0)^2+c_0(\alpha_1-l_1)^2}. \tag{1.17}$$

We further write

$$p^l(\theta)=\overline{p}^l+\gamma^l\left(\frac{\theta(\alpha_0-l_0)}{c_0}+\frac{\theta(\alpha_1-l_1)}{c_1}\right), \tag{1.18}$$

where

$$\overline{p}^l = E\left[p^l(\theta)\right] = \gamma^l\left(\frac{b_0(\alpha_0 - l_0)}{c_0} + \frac{b_1(\alpha_1 - l_1)}{c_1} - l\right). \tag{1.19}$$

The operation of the non-sterilized system is straightforward; just fix the number of auctioned permits to a predetermined level. As for the sterilized system, we suggest that the agency operates the market with the help of so-called discount coupons. A discount coupon is not a permit, but the holder of a coupon can exchange it for a permit. Within that system, a polluting unit receives the emission threshold as coupons upon registering as an active firm. Later on, the unit uses coupons in the permits auctions. That is, after the unit has bid an amount of permits in the auctions, the discount coupons will be deducted from the total bill. As far as the price of permits is positive, every participating firm will use every coupon in the auction. At the same time, the agency is effectively auctioning the endowment of aggregate permits, so it controls the aggregate level of emissions.[4]

1.3 Social Welfare Optimization

1.3.1 Efficient Policy

We start our study of efficiency by setting $\theta = 0$. Based on the discussion of the polluting industry above, the total benefits are

$$B = \int_0^{\lambda_0} B_0 d\lambda + \int_0^{\lambda_1} B_1 d\lambda, \tag{1.20}$$

where B_0 and B_1 are defined in Equation (1.1). A question of particular interest is the emission allocation between the sectors. Among various allocations, the efficient allocation yields maximum total benefits for a given level of emissions. We then maximize benefits (Equation (1.20)) given the emissions (Equation (1.6)). If we denote the Lagrange multiplier by μ, the efficient allocation satisfies the conditions

$$B_i(\lambda_i) - \mu\alpha_i = 0,$$

[4]There are other examples in the literature, where the operation of the permit system is refined to account for some extra needs. These refinements include the system of rental emission permits by Collinge and Oates [8] and the hybrid system by Roberts and Spence [62].

or, accordingly, the conditions

$$\frac{B_i(\lambda_i)}{\alpha_i} = \mu, \tag{1.21}$$

where $i = 0, 1$. The right-hand sides of Equations (1.21) are independent of i, so we have

$$\frac{B_0(\lambda_0^e)}{\alpha_0} = \frac{B_i(\lambda_1^e)}{\alpha_1}.$$

Since the $B_i(\lambda_i)$ is the benefit of the cut-off unit in sector i, this condition says that the average benefits of emissions should be the same upon efficient allocation.

The choices in the polluting sectors are driven by the profits. By Equation (1.2), the sector i response satisfies

$$B_i(\lambda_i) = s(\alpha_i - l_i). \tag{1.22}$$

In particular, if

$$\frac{B_0(\lambda_0)}{\alpha_0} = \frac{B_1(\lambda_1)}{\alpha_1},$$

then we have $\lambda_i^e = \lambda_i$. To satisfy this condition, we set

$$\frac{s(\alpha_0 - l_0)}{\alpha_0} = \frac{s(\alpha_1 - l_1)}{\alpha_1}, \tag{1.23}$$

or

$$\frac{\alpha_0 - l_0}{\alpha_0} \equiv \frac{\alpha_1 - l_1}{\alpha_1} \equiv \omega. \tag{1.24}$$

The relationship between the thresholds l_0 and l_1 ought to be satisfied upon efficient allocation.

The efficiency is clearly satisfied, if $l_0 = l_1 = 0$. This is a standard result in the entry-exit literature, and it states that the “polluter pays." Accordingly, the agency should not distribute free allocations. Papers like the one by Pezzey [54] discuss this result further. However, note that the efficient rule in Equation (1.24) allows for strictly positive values of l_0 and l_1 as well. In every case, it holds that $0 \leq \omega \leq 1$.

Every allocation, whether efficient or inefficient, implies a certain benefit function. It is a relation between aggregate benefits and aggregate emissions. By construction, the benefits in efficient policy should be as high as possible for every level of e. If we denote these benefits by $B(e)$, then a corollary for our late efficiency maximization provides us with the condition

$$\frac{dB(e)}{de} = \mu.$$

By Equations (1.1), (1.6), and (1.21), we can write

$$e = \frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - \mu\frac{b_0\alpha_0^2}{c_0} + \mu\frac{b_1\alpha_1^2}{c_1},$$

so

$$\mu = \gamma\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - e\right),$$

where

$$\gamma = \frac{c_0c_1}{c_1(\alpha_0)^2 + c_0(\alpha_1)^2}. \tag{1.25}$$

Consequently, the abatement cost function is

$$B(e) = \int \mu(e)de.$$

In our case, it is a quadratic function of emissions, so marginal benefit function is linear. The literature calls μ the marginal abatement function, so γ is the slope of the marginal abatement function.

Our model is similar to Weitzman since marginal benefits depend linearly on emissions. At the same time, we are interested in inefficient allocations. We should then clarify how the inefficiency affects the benefit function.

1.3.2 Optimal Policies

1.3.2.1 The Tax Implementations

Let us bring the uncertainty and emission damages into our analysis. The damages of emissions depend on the aggregate amount of emissions and there is no uncertainty in the quadratic damage function. That is, the damages are

$$D(e) = \frac{d}{2}e^2,$$

where $d > 0$. We will next derive optimal policies and start with the first-best policy. In the first-best policy, the environmental agency can choose every policy variable τ, l_0, and l_1, so the optimization amounts to the following exercise:

$$\underset{\tau,l_0,l_1}{Max}\ EW = E\left[\int_0^{\lambda_0(\tau,l_0,\theta)} B_0 d\lambda + \int_0^{\lambda_1(\tau,l_1,\theta)} B_1 d\lambda\right]$$
$$-ED(e(\lambda_0(\tau,l_0,\theta),\lambda_1(\tau,l_1,\theta))$$

such that

$$e = \lambda_0(\tau,l_0,\theta)\alpha_0 + \lambda_1(\tau,l_1,\theta)\alpha_1. \tag{1.26}$$

The first order conditions (see Appendix A.1) are

$$\tau - \frac{\gamma^l}{\gamma^L} dE[e] = 0, \tag{1.27}$$

$$\tau - \frac{\alpha_0}{(\alpha_0 - l_0)} dE[e] = 0, \tag{1.28}$$

and

$$\tau - \frac{\alpha_1}{(\alpha_1 - l_1)} dE[e] = 0. \tag{1.29}$$

Clearly, the last two conditions imply that

$$\frac{\alpha_0 - l_0}{\alpha_0} = \frac{\alpha_1 - l_1}{\alpha_1}$$

at the optimum, so (see Equation (1.24)) the optimal allocation should be efficient as well. As for the optimal tax rate, we note that (by Equations (1.12) and (1.17)),

$$\frac{\gamma^l}{\gamma^L} = \frac{c_1(\alpha_0 - l_0)\alpha_0 + c_0(\alpha_1 - l_1)\alpha_1}{c_1(\alpha_0 - l_0)^2 + c_0(\alpha_1 - l_1)^2} = \frac{c_1\left(\frac{\alpha_0 - l_0}{\alpha_0}\right)\alpha_0^2 + c_0\left(\frac{\alpha_1 - l_1}{\alpha_1}\right)\alpha_1^2}{c_1\left(\frac{\alpha_0 - l_0}{\alpha_0}\right)^2\alpha_0^2 + c_0\left(\frac{\alpha_1 - l_1}{\alpha_1}\right)^2\alpha_1^2}$$

and after incorporating the efficiency rule (Equation (1.24)),

$$\frac{\gamma^l}{\gamma^L} = \frac{1}{\omega}. \tag{1.30}$$

Thus, the first-best policy is a tax rate that satisfies

$$\tau = \frac{dE[e]}{\omega}. \tag{1.31}$$

In summary, we do not have one but two optimal rates. In the first case, $l_0 = l_1 = 0$, so $\omega = 1$, and

$$\tau^0 = dE[e].$$

This is a standard policy rule that recommends setting the tax equal to the expected marginal damages. Alternatively, $0 < \omega < 1$, so

$$\tau^e = \frac{\tau^0}{\omega}. \tag{1.32}$$

Clearly, $\tau^e > \tau^0$.

It should be intuitively clear that all the differences between optimal implementations are only nominal. Formally, we show in Appendix A.2 that $e(\tau^0, \theta) = e(\tau^e, \theta)$. In other words, every optimal implementation will yield the same level of emissions *ex-post*. Furthermore, as far as

$$E[e(\tau, \theta)] = \alpha_0\left(\frac{b_0}{c_0}\right) + \alpha_1\left(\frac{b_1}{c_1}\right) - \omega\frac{\tau}{\gamma},$$

the optimal level of expected emissions (in terms of the parameters of the model) satisfies

$$E[e(\tau, \theta)] = \alpha_0\left(\frac{b_0}{c_0}\right) + \alpha_1\left(\frac{b_1}{c_1}\right) - \frac{d}{\gamma}E[e(\tau, \theta)].$$

If we denote the optimal level by e^*, it satisfies

$$e^* = \frac{\gamma}{\gamma + d}\left(\alpha_0\left(\frac{b_0}{c_0}\right) + \alpha_1\left(\frac{b_1}{c_1}\right)\right). \tag{1.33}$$

1.3.2.2 The Permit Implementations

In an efficient permit implementation, the agency auctions all or only part of the permits off. In the latter case, the agency has to make sure that the relation $\frac{l_0}{l_1} = \frac{\alpha_0}{\alpha_1}$ between the thresholds holds (see Equation (1.24)). In calculating the optimal policy, we will follow Montero [46] by applying the expected price as the choice variable. Otherwise, we optimize in the same manner as we did above in the tax policy. We choose the optimal expected price in the sterilized system ($\overline{p}^L$ in Equation (1.14)) and in the non-sterilized system ($\overline{p}^l$ in Equation (1.19)). In practice, the regulator does not choose the prices directly but supplies an appropriate number of permits to the market. Note further that we have two cases to study within sterilized and non-sterilized systems. These follow since both zero and positive thresholds can implement the optimality in both systems.

We present the derivations in Appendix A.3. We denote the optimal prices under zero and positive thresholds by p^0 and p^e, respectively. Accordingly, if every permit is auctioned off, then

$$\overline{p}^L = \overline{p}^l = p^0.$$

Alternatively, if a part of the endowment is given for free, then

$$\overline{p}^L = \overline{p}^l = p^e.$$

It also holds that

$$p^e = \frac{p^0}{\omega}. \tag{1.34}$$

Moreover, the aggregate number of permits in every system equals $L = e^*$. The supply of auctioned permits in the non-sterilized system is $l = \omega L < L$.

The optimal permit policies restate our earlier findings with the optimal tax policies. The prices reflect nominal differences, not real ones. In particular, as implementations are optimal, the aggregate permit endowment is always chosen to satisfy $L = e^*$, where e^* is the optimal level of emissions in the tax system (Equation

(1.33)). As for the nominal effects, the introduction of free permits becomes transformed into the permit price. As $0 < \omega < 1$, then $p^e > p^0$, so the subsidization of firms will increase the permit price. The rise in the permit price is a sort of a wealth effect. Every firm in the market becomes more profitable after the thresholds are distributed among the units in the polluting industry. The rise in profitability will shift demand and, consequently, the permit price upwards. Note also how the amount of auctioned permits is smaller than the number of aggregate permits ($l < L$). This simply reflects the fact that part of the permits is given out for free.

1.3.3 Second-Best Policy

We assume next that the choices of thresholds (l_0 and l_1) lie outside the authority of the environmental agency. However, the strictness of an environmental policy is still under the control of it. In choosing the strictness, the agency maximizes the expected social welfare given the values of l_0 and l_1. In other words, it uses only one of the first-order conditions, namely, the condition written in Equation (1.27). Then

$$-\frac{\tau}{\gamma^l} + \frac{d}{\gamma^L} E[e] = 0.$$

We insert $\lambda_0(\tau, l_0, \theta)$ and $\lambda_1(\tau, l_1, \theta)$ into Equation (1.26), so

$$E[e] = \frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1} - \frac{\tau}{\gamma^L}, \tag{1.35}$$

and we have the (restricted, second-best) optimum rate as

$$\tau^s = \frac{d\gamma^L \gamma^l}{[\gamma^L]^2 + d\gamma^l} \left(\alpha_0 \left(\frac{b_0}{c_0} \right) + \alpha_1 \left(\frac{b_1}{c_1} \right) \right). \tag{1.36}$$

Inserting the second-best tax rate back to Equation (1.35) gives us the expected second-best level of emissions as

$$e^s = \left(\frac{[\gamma^L]^2}{[\gamma^L]^2 + d\gamma^l} \right) \left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1} \right). \tag{1.37}$$

It is interesting to know how the second-best policy compares to the first-best. Toward that end, we induce efficiency into the maximization by setting $l_0 = l_1 = 0$.

By Equations (1.31) and (1.33), the optimal tax rate[5] (denoted by τ^0) becomes

$$\tau^0 = \frac{d\gamma}{\gamma + d}\left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1}\right). \tag{1.38}$$

We have:

i. By assuming efficiency, the second-best price and quantity become first-best.

ii. $\tau^s > \tau^0$.

ii. $e^s > e^*$.

We prove and discuss these claims in Appendix A.4.

We can derive the second-best permit policy along the same basic guidelines as we derive the first-best policy. However, as far as the implementation is always subsidized, only one expected unit price is needed in the sterilized and non-sterilized systems. The sterilized system should fix the total permit endowment in such a way that the condition

$$\overline{p}^L = \tau^s$$

holds. Alternatively, the non-sterilized system should fix the auctioned permit endowment in such a way that the condition

$$\overline{p}^l = \tau^s$$

holds. In every case, the aggregate number of permits should be equal to e^s.

1.3.4 Graphical Illustration of the Policies

Let us illustrate the policies just derived with the help of a figure. Up to this point, we have developed our model in terms of the price variable. However, graphical illustrations of the calculations are far more illustrative if they include the quantity variable, that is, the emissions as well. Intuitively, this approach allows the simultaneous incorporation of the marginal benefits, marginal damages, and the two policy parameters into the figure—the tax rate and the permit endowment.

[5]We hope that our choice to use the same notation to symbolize both efficient and optimal outcomes does not bother the reader too much. After all, every optimal outcome is an efficient outcome as well.

Consider then Figure 1.1, where the vertical axis displays the price and the horizontal axis represents emissions. In general, the following rule governs the switch between quantities and prices:

$$e-\left(\alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)-\frac{c_1(\alpha_0-l_0)\alpha_0+c_0(\alpha_1-l_1)\alpha_1}{c_0c_1}s\right)\equiv 0. \qquad (1.39)$$

This follows, as we expand the equation $e=\lambda_0(\theta)\alpha_0+\lambda_1(\theta)\alpha_1$. By using Equation (1.39) (together with the definition of γ^L in Equation (1.12)), we write

$$s(e;\theta)=\gamma^L\left(\alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)\right)-\gamma^L e \qquad (1.40)$$

and call it *the price function*. Next, the benefits (Equation (1.20)) expanded yield

$$\begin{aligned}B&=\int_0^{\lambda_0(\theta)}B_0 d\lambda+\int_0^{\lambda_1(\theta)}B_1 d\lambda\\&=\frac{(b_0+\theta)^2}{2c_0}+\frac{(b_1+\theta)^2}{2c_1}-\left[\frac{1}{2c_0}(\alpha_0-l_0)^2+\frac{1}{2c_1}(\alpha_0-l_0)^2\right]s^2,\end{aligned}$$

or, by Equation (1.17),

$$B=\frac{(b_0+\theta)^2}{2c_0}+\frac{(b_1+\theta)^2}{2c_1}-\frac{s^2}{2\gamma^l}.$$

Insert the price function $s(e;\theta)$ into this, arrange the terms, and write

$$B(e;\theta)=B_0(\theta)+B_1(\theta)e-\frac{(\gamma^L)^2}{2\gamma^l}e^2,$$

where

$$B_1(\theta)=\frac{(\gamma^L)^2}{\gamma^l}\left(\alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)\right).$$

Thus, the marginal benefits are

$$\frac{dB(e;\theta)}{de}=B_1(\theta)-\frac{(\gamma^L)^2}{\gamma^l}e=B_1(\theta)-\gamma\rho e.$$

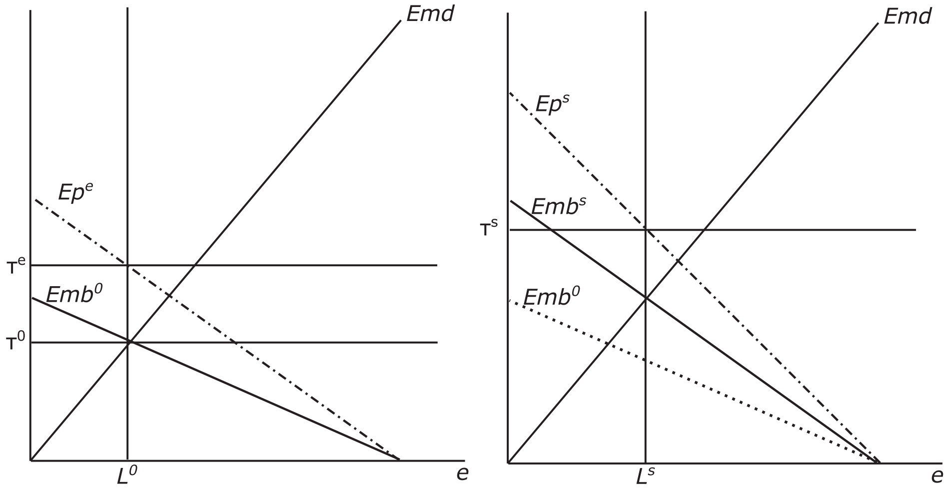

Figure 1.1 Implementation of the First-Best Policy (a); Implementation of the Second-Best Policy (b)

In particular, we will show below (see Equation (1.58)) that

$$\rho > 1$$

as long as allocation is inefficient. In case of efficiency, we have $\rho = 1$.

In Figure 1.1, we employ three distinct lines to describe the determination of the various optimal policies. We have lines

$$Ep = Es(e;\theta) = \gamma^L \left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - e \right), \tag{1.41}$$

$$Emb = E\left[\frac{dB(e;\theta)}{de} \right] = \gamma\rho \left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - e \right), \tag{1.42}$$

and

$$Emd = de.$$

The lines correspond to expected price function, expected marginal benefit function, and expected marginal damage function, respectively. In every policy, we can determine the optimal quantity by the following rule:

$$Emb = Emd. \tag{1.43}$$

However, further policy details are case-specific.

Consider first the determination of the first-best policy. A conspicuous feature in the policy is that the agency can implement it using zero or strictly positive thresholds. As far as implementations are efficient in both cases, we have $\rho = 1$. We denote the curves by Emb^0 (zero thresholds) and Emb^e (positive thresholds), so with the reference to Equation (1.42), it holds that

$$Emb^0 = Emb^e = \gamma\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - e\right).$$

On the other hand, by Equation (A.4 in Appendix A.2), we write Equation (1.41) as

$$Ep = \frac{\gamma}{\omega}\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - e\right).$$

Thus, if thresholds are equal to zero, $\omega = 1$ and $Ep^0 = Emb^0$. In this case, the price and the marginal benefit functions will overlap. This means that the optimal price–quantity pair is determined by the marginal benefit function. Alternatively, the thresholds are strictly positive. Then, $0 < \omega < 1$, so $Ep^e \neq Emb^e$. In this last case, we say that *the price is separately determined*.

We can demonstrate these policies in Figure 1.1(a). As for the optimal expected emissions, it is determined by the rules that

$$Emb^0 = Emb^e = Emd.$$

We denote the corresponding optimal expected level by L^0 in the figure. As for the optimal prices, if $l_0 = l_1 = 0$, then the optimal price satisfies $\tau^0 = Ep^0(L^0) = Emb^0(L^0)$. Alternatively, if either $l_0 > 0$ or $l_1 > 0$ or both, then the optimal price satisfies $\tau^e = Ep^e(L^0) > Emb^e(L^0)$. We discussed above that $\tau^e > \tau^0$.

Consider next the determination of the second-best policy. First, as the policy uses subsidization, then

$$Ep^s \neq Emb^s,$$

so the price is separately determined. Second, as the policy is inefficient, then $\rho > 1$ and

$$Emb^s \neq Emb^0.$$

Thus, the marginal benefit function (Emb^s) does not overlap with either the price function or the efficient marginal function.

The second-best optimal price–quantity pair (τ^s, L^s) is illustrated in Figure 1.1(b). The second-best expected emission is determined by the rule

$$Emb^s = Emd.$$

The corresponding second-best price satisfies

$$\tau^s = Ep^s(L^s).$$

In particular, as we discussed above (see also the results in Appendix A.4), we have $L^s > L^0$ and $\tau^s > \tau^0$.

1.4 Choosing Between the Instruments

1.4.1 Prices and Quantities

We will start our analysis of instrument choice by writing some new helpful notation. In particular, we like to rewrite the price process in a general form as

$$s = Es + \gamma^L \left[R_0(s) \left(\frac{\alpha_0}{c_0} \right) \theta + R_1(s) \left(\frac{\alpha_1}{c_1} \right) \theta \right] \tag{1.44}$$

for various $R_0(s)$ and $R_1(s)$. The unit price inside the parentheses indicates that the factor is specific to the type of implementation. The two price processes in Equations (1.13) and (1.18) can now be written as

$$p^L(\theta) = \overline{p}^L + \gamma^L \left(\frac{\alpha_0}{c_0}\theta + \frac{\alpha_1}{c_1}\theta \right) \tag{1.45}$$

and

$$p^l(\theta) = \overline{p}^l + \gamma^L \left[R_0(p^l) \left(\frac{\alpha_0}{c_0} \right) \theta + R_1(p^l) \left(\frac{\alpha_1}{c_1} \right) \theta \right], \tag{1.46}$$

respectively, where

$$R_i(p^l) = \frac{\gamma^l}{\gamma^L} \frac{(\alpha_i - l_i)}{\alpha_i},$$

and $i = 0, 1$. Thus, we have $R_0(p^L) = R_1(p^L) = 1$. The price in Equation (1.45) is sterilized, while in Equation (1.46) it is not. We also find it convenient to define

$$k \equiv \frac{(\alpha_1 - l_1)}{(\alpha_0 - l_0)},\ u \equiv \frac{c_0}{c_1},\ \text{and } a \equiv \frac{\alpha_1}{\alpha_0}, \tag{1.47}$$

so we will switch to relative parameters.

Using these definitions, and after some manipulation, we may further write

$$R_0(p^l) = \frac{1+uka}{1+uk^2} \text{ and } R_1(p^l) = \frac{k}{a}R_0(p^l). \tag{1.48}$$

We state (without giving any formal proof) that the following characteristics for permit price $p^l(\theta)$ (Equation (1.46)) hold:

i. $R_0 \geq 0, R_1 \geq 0$, and $R_0 + R_1 > 0$
ii. If $k \longrightarrow \infty$, then $R_0 = 0$, $R_1 = 1$
iii. If $k = 0$, then $R_0 = 1$, $R_1 = 0$
iv. If $k = a$, then $R_0 = R_1 = 1$
v. If $k < a$, then $R_0 > 1$, $R_1 < 1$
vi. If $k > a$, then $R_0 < 1$, $R_1 > 1$.

Note in particular that the value $k = a$ refers to an efficient allocation. Table 1.1 summarizes the various values of $R_0(s)$ and $R_1(s)$ under different implementations.

Table 1.1 The R-Factors for the Various Instruments

	$R_0(s)$	$R_1(s)$
τ	0	0
p^L	1	1
p^l	≥ 0	≥ 0

It is also a straightforward task to write the aggregate level of emissions as

$$e(s) = \bar{e} + (1 - R_0(s))\frac{\alpha_0}{c_0}\theta + (1 - R_1(s))\frac{\alpha_1}{c_1}\theta, \tag{1.49}$$

where $\bar{e}$ is again independent of θ. Finally, we consider the various differences in prices and quantities in Table 1.2.[6] We calculate the price differences in Appendix A.5, Part I, while we calculate the various quantity differences in Appendix A.5, Part II.

Table 1.2 Various Differences in Prices and Quantities Between Different Instruments

$p^L(\theta)-p^l(\theta)=\frac{1}{(\alpha_0-l_0)}\frac{u(a-k)(1-k)}{(1+uka)(1+uk^2)}\theta$
$e(p^l(\theta))-e(\tau(\theta))=-\frac{1}{\gamma^L}\frac{1}{(\alpha_0-l_0)}\frac{1+uk}{1+uk^2}\theta$
$e(p^L(\theta))-e(\tau(\theta))=-\frac{c_1\alpha_0+c_0\alpha_1}{c_1c_0}\theta$
$e(p^L(\theta))-e(p^l(\theta))=\frac{1}{\gamma^L}\frac{(a-k)(k-1)}{(\alpha_0-l_0)}\frac{u}{(1+uk^2)(1+uak)}\theta$

A prominent feature concerns the path of the aggregate non-sterilized emissions, $e(p^l)$. By Equation (1.49), as long as either $R_0(p^l)$ or $R_1(p^l)$ is different from one, the emissions are not fixed but differ from the expected level of emissions $\overline{e}$. However, as far as efficiency implies that $R_0(p^l)=R_1(p^l)=1$, the efficient implementation of the quantity policy inevitably yields a fixed level of emissions. Regarding the sterilized emissions, $e(p^L)$, Equation (1.49) merely confirms that the aggregate emissions do not fluctuate under the sterilized system. Another important observation concerns the behavior of taxed emissions. By construction, the influence of the threshold allocation (l_0, l_1) is channeled entirely through factors $R_0(s)$ and $R_1(s)$. We then see from Equation (1.49) that the aggregate emission variation under taxes is totally independent of the allocation (l_0, l_1).

If $R_0(p^l)\neq 1$ and $R_1(p^l)\neq 1$, then the non-sterilized permit system creates a new type of aggregate variation among the instruments. Interestingly, when compared to the studies employing efficient instruments, there is presumably a new kind of trade-off present between the quantity and the price instruments. The non-sterilized permits may gain a new advantage, as the aggregate quota is no longer fixed. At the same time, there arises some (most likely harmful) effects as the permits are traded using inefficient trading ratios. This last observation is partly confirmed in Appendix A.5, Part III, where we concentrate on sector-specific allocations. Whether a sterilized or non-sterilized permit system, the sector-specific emissions allocations clearly

[6] A minor technical note: The various differences in the table depend on the multiplier $\frac{1}{(\alpha_0-l_0)}>0$. We could have derived the formulas in terms of $\frac{1}{(\alpha_1-l_1)}$ instead, and the results would have remained the same.

differ from efficient patterns. This observation also means that the more traditional quantity instrument—the sterilized system—undergoes changes. In particular, we may conclude that the allocation $(e_0(p^L), e_1(p^L))$ is not an efficient way to distribute the totality of $\overline{e}$. Instead, the sector-specific allocation under taxation, confirms our earlier conclusion: the emission variation is totally independent of the allocation (l_0, l_1).

Regarding the permit prices, the equality $p^l(\theta) = p^L(\theta)$ holds under two separate subsidy profiles. First, it holds whenever $k = a$. This means that the two prices are identical under efficient subsidization. Second, the equality between the prices also holds when $k = 1$. This occurs if an absolute subsidization rule $(\alpha_1 - l_1) = (\alpha_0 - l_0)$ holds. However, this rule does not yield efficiency.[7] Looking at the lower part of the table, if $k = a$ or $k = 1$, we have $e(p^l(\theta)) = e(p^L(\theta))) = \overline{e}$. Thus, the system of non-sterilized permits is able to stabilize the emissions by utilizing two distinct permit thresholds. However, at the same time, the fixed level of emissions differs between subsidy profiles. If we let e^* denote the efficient emissions, then $\overline{e} = e^*$ whenever $k = a$. Instead, if $k = 1$, we have $\overline{e} \neq e^*$. As for the difference $e(p^l(\theta))$—$e(\tau(\theta))$, it can be shown that the non-sterilized system cannot induce the same regulated emissions as the tax system does. Consequently, as taxed emissions (by definition) correspond to a fixed price level, the system of non-sterilized permits is not able to stabilize the permit price.

The content of Table 1.2 provides some preliminary intuition for the forthcoming studies of instrument choice. One observation concerns the nature of the critical points $k = a$ and $k = 1$. We just saw how the price level and the emission level remain fixed at these particular policies. Referring to our upcoming definitions, we say that the so-called volume effect vanishes there. On the other hand, at $k = a$, the implementation is efficient, while at $k = 1$, it is not. Interestingly, from an efficiency point of view, the system of non-sterilized permits produces both fake ($k = 1$) and *bona fide* ($k = a$) critical points. In future discussion, we claim that the so-called cost effect vanishes at $k = a$, while it does not vanish at $k = 1$.

[7] More specifically, let $p^e(\theta)$ denote the permit price under efficient subsidization. Then, if $k = a$, we have $p^l(\theta) = p^L(\theta) = p^e(\theta)$. Instead, if $k = 1$, we have $p^l(\theta) = p^L(\theta) \neq p^e(\theta)$.

1.4.2 Comparative Advantage

Weitzman [84] defines the comparative advantage between instrument I and J as

$$\Delta(I,J) = E\left[(B(I) - D(e(I))) - (B(J) - D(e(J)))\right].$$

Under our framework (use cut-off units λ_i in Equations (1.4)),

$$B(s) = \int_0^{\lambda_0} B_0 d\lambda + \int_0^{\lambda_1} B_1 d\lambda = \left[\frac{(b_0+\theta)^2}{2c_0} + \frac{(b_1+\theta)^2}{2c_1}\right] - \left[\frac{(s(\theta))^2}{2\gamma^l}\right] \tag{1.50}$$

and

$$\begin{aligned} D(e(s)) &= \frac{d}{2}(e(s))^2 = \frac{d}{2}(\int_0^{\lambda_0} \alpha_0 d\lambda_0 + \int_0^{\lambda_1} \alpha_1 d\lambda_1)^2 \\ &= \frac{d}{2}\left(\alpha_0\left(\frac{b_0+\theta}{c_0}\right) + \alpha_1\left(\frac{b_1+\theta}{c_1}\right) - \frac{s(\theta)}{\gamma^L}\right)^2. \end{aligned} \tag{1.51}$$

In what follows, we find it helpful to operate in terms of variances. We refer to Appendix A.6 as we rewrite the comparative advantage between instruments I and J as

$$\Delta(I,J) = \frac{Var(J) - Var(I)}{2}\left[\frac{1}{\gamma^l} - d\left(\frac{Var(e(I)) - Var(e(J))}{Var(J) - Var(I)}\right)\right]. \tag{1.52}$$

Let us fix the order of instruments for a moment, so that $Var(J) > Var(I)$. If instrument J has a higher variance in quantities as well, then $\Delta(I,J) > 0$, and the agency will always choose instrument I. Thus, in a meaningful comparison, we must then have $Var(e(I)) > Var(e(J))$. Note also that the conditions $Var(I) = Var(e(J)) = 0$ simultaneously hold in the traditional Weitzman comparison so they automatically ensure a proper analysis of comparative advantage.[8] We further denote

$$v \equiv v(I,J) = \frac{Var(I)}{Var(J)}, \tag{1.53}$$

so

$$\Delta(I,J) = Var(J)\frac{1-v}{2}\left[\frac{1}{\gamma^l} - d\left(\frac{Var(e(I)) - Var(e(J))}{Var(J) - Var(I)}\right)\right]. \tag{1.54}$$

1.4.3 The Volume Effect and Cost Effect

We denote

$$\Theta \equiv \Theta(I,J) = \left(\gamma^L\right)^2 \frac{Var(e(I)) - Var(e(J))}{Var(J) - Var(I)} \tag{1.55}$$

and rewrite Equation (1.54) as

$$\Delta(I,J) = Var(J)\frac{1-v}{2}\left(\frac{1}{\gamma^l} - \Theta\frac{d}{(\gamma^L)^2}\right). \tag{1.56}$$

Let us call factor Θ the volume effect. In principle, the volume effect can take any values. The negative values in particular imply that agency should always choose instrument I.[9] We also say that if $\Theta > 1$ ($\Theta < 1$), then the volume effect will favor instrument J (instrument I). If $\Theta = 1$, then we say that the volume effect disappears.

We would like to study $\Delta(I,J)$ in terms of the fundamentals γ and d. To do this, we write the Equation (1.56) as

$$\Delta(I,J) = Var(J)\frac{1-v}{2}\frac{\rho}{(\gamma^L)^2}\left(\gamma - \frac{\Theta}{\rho}d\right), \tag{1.57}$$

where

$$\rho \equiv \frac{\left(\gamma^L\right)^2}{\gamma\gamma^l}.$$

[8] In addition to benefit uncertainty, Weitzman [84] assumes an uncertainty variable that shifts the marginal damages. However, he further assumes that different uncertainties are independent. This assumption means that the comparative advantage measure becomes independent of damage uncertainty. In our study, had we assumed additive damage uncertainty and independent uncertainties, our results would have remained the same. In this important respect, certain damages induce no loss of generality. We briefly illustrate this issue in Appendix A.7.

[9] In this case, it holds that $0 < v < 1$.

We call factor ρ the cost effect. The cost effect arises in the instrument choice whenever regulation becomes inefficient. In terms of the auxiliary variables k, a, and u (see Equations (1.47)), we can write the cost effect as

$$\rho = 1 + \frac{u\,(k-a)^2}{(1+uak)^2}. \tag{1.58}$$

As $u > 0$, and if we let $k \neq a$, then

$$\rho > 1.$$

Clearly, if $k = a$, then $\rho = 1$. Recall that the condition $k = a$ implies efficiency.

In Equation (1.57), the term ρ lies both inside and outside of the parenthesis. Since $\rho \geq 1$, the term ρ outside only magnifies the size of the measure Δ, so it does not affect the choice of the instrument. If subsidization is inefficient, it holds that $0 < \frac{1}{\rho} < 1$. We then say that the cost effect favors instrument I. Note also that (by Equation (1.58)) the size of ρ increases, as k moves away from a. Since $k = a$ implies efficiency, then the cost effect monotonically increases (that is, favors more and more instrument I) as we move further away from the efficient solution.[10] Finally, if $k = a$, then $\rho = 1$, and we say that the cost effect disappears.

Above, we have a particular order between instruments in the advantage formulas. More generally, we may state that

i. $\Theta(I,J) = \Theta(J,I)$.
ii. The cost effect favors the instrument with the lowest variance in price.
iii. Let $\Theta > 1$ ($\Theta < 1$). The volume effect favors the instrument with the highest (lowest) variance in price.

We discuss these properties in Appendix A.8.

1.4.4 Prices Versus Quantities with Fixed Emissions

1.4.4.1 The Disappearance of the Volume Effect

In this section, we start our review of instrument choice by assuming that $I = \tau$. Thus, the price instrument—the environmental tax—is a candidate in the policy im-

[10] We do not develop the measurement of inefficiency beyond this notion.

plementation. We write the various variances in terms of R_0 and R_1 as

$$Var(s) = \left(\gamma^L\right)^2 \sigma^2 \left[R_0(s)\left(\frac{\alpha_0}{c_0}\right) + R_1(s)\left(\frac{\alpha_1}{c_1}\right)\right]^2 \tag{1.59}$$

and

$$Var(e(s)) = \sigma^2 \left[(1 - R_0(s))\frac{\alpha_0}{c_0} + (1 - R_1(s))\frac{\alpha_1}{c_1}\right]^2 \tag{1.60}$$

for instrument s. Specifically, (by Table 1.1) we have $R_0(\tau) = R_1(\tau) = 0$, so the volume effect (Equation (1.55)) between the tax and quantity instrument J becomes

$$\begin{aligned} \Theta(\tau, J) &= \left(\gamma^L\right)^2 \frac{[Var(e(\tau) - Var(e(J))]}{Var(J) - Var(\tau)} \\ &= \frac{\left[\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}\right]^2 - \left[(1 - R_0(J))\frac{\alpha_0}{c_0} + (1 - R_1(J))\frac{\alpha_1}{c_1}\right]^2}{\left[R_0(I)\left(\frac{\alpha_0}{c_0}\right) + R_1(I)\left(\frac{\alpha_1}{c_1}\right)\right]^2}. \end{aligned} \tag{1.61}$$

By closer inspection, whenever $R_0(J) = R_1(J) = 1$, then $\Theta(\tau, J) = 1$ and the volume effect disappears. This occurs under the sterilized permit system, namely, under the system that fixes the level of emissions.

1.4.4.2 Efficient Implementations

In prices versus quantities, assume first that the regulation utilizes efficient instruments. Referring to our discussion above, it now holds that $\rho = 1$ and $\Theta = 1$. Both cost and volume effects will disappear (and $v = 0$ by Equation (1.53)), so the comparative advantage (Equation (1.57)) becomes

$$\Delta(\tau^i, p^i) = Var(p^i)\frac{\omega^2}{2\gamma^2}(\gamma - d), \tag{1.62}$$

where $i = 0, e$. If $i = 0$, the thresholds are equal to zero, and if $i = e$, the thresholds are strictly positive.

Whenever $i = 0$, then (by Equation (1.24)) $\omega = 1$. Thus,

$$\Delta(\tau^0, p^0) = \frac{Var(p^0(\theta))}{2\gamma^2}(\gamma - d).$$

Alternatively, we have $i = e$. By Equation (1.34), $p^e(\theta) = \frac{p^0(\theta)}{\omega}$. Insert this information into Equation (1.62), so it follows that

$$\Delta(\tau^e, p^e) = \frac{Var(p^0(\theta))}{2\gamma^2}(\gamma - d) = \Delta(\tau^0, p^0). \tag{1.63}$$

This is the original Weitzman [84] result calculated in an augmented framework. Prices are preferred over quantities if the slope of the marginal benefit function (γ) exceeds the slope of the marginal damage function (d). We regard this as a fundamental result. The choice depends on the slope parameters γ and d that belong to the social welfare function. This result also shows that the size of the measure Δ does not depend on the type of efficient implementation. That is, whether zero or strictly positive thresholds are applied, the measure Δ remains the same.

1.4.4.3 Inefficient Implementation

We assume next that the thresholds l_0 and l_1 do not satisfy efficiency. However, we assume a fully binding aggregate permit supply, which implies a fixed level of emissions. We have[11]

$$\Theta(\tau, p^L) = 1, \tag{1.64}$$

so, by Equation (1.57), we may write

$$\Delta(\tau, p^L(\theta)) = \frac{\rho}{2}\frac{Var(p^L(\theta))}{(\gamma^L)^2}(\gamma - \frac{1}{\rho}d).$$

If we apply the specific relation between $Var(p^L(\theta))$ and $Var(p^0(\theta))$, we may also write

$$\Delta(\tau, p^L(\theta)) = \rho\frac{Var(p^0(\theta))}{2\gamma^2}(\gamma - \frac{1}{\rho}d). \tag{1.65}$$

In general, the overall effect that inefficient subsidization induces is $\frac{\Theta}{\rho}$. Here, the volume effect disappears, so the total effect consists only of the cost effect. The cost effect will favor the tax instrument. This is as expected. We discuss above that the cost effect favors the instrument with the lowest variance. In fact, as $Var(\tau) = 0$, the cost effect is very favorable to taxes. We may also say that inefficient subsidization

[11] Note that the instrument from now on is chosen in a second-best framework. We apply subscript s in the policy variables above to differentiate them from their first-best counterparts. However, to minimize symbolization, we no longer apply subscript s below.

hurts the socially favorable trading ratios in permit markets.

1.4.5 Graphical Illustration of the Cost Effect

We illustrate the previous comparative advantage graphically in Figure 1.2. We employ a presentation, where we write the benefits and damages in terms of the quantity, that is, in terms of the emissions. In Figure 1.2, the marginal benefits are

$$mb(e;\theta)=\gamma\rho\left(\frac{b_0\alpha_0}{c_0}+\frac{b_1\alpha_1}{c_1}+\left(\frac{c_0\alpha_1+c_1\alpha_0}{c_1c_0}\right)\theta-e\right), \tag{1.66}$$

while the price is determined according to the price function

$$s=s(e;\theta)=\gamma^L\left(\frac{b_0\alpha_0}{c_0}+\frac{b_1\alpha_1}{c_1}+\left(\frac{c_0\alpha_1+c_1\alpha_0}{c_1c_0}\right)\theta-e\right). \tag{1.67}$$

We refer the reader back to an earlier section where we explain the meanings of these expressions.[12] Both the marginal benefit and the price functions display the cost effect. As far as $\rho=1$ and $\gamma^L=\frac{1}{\omega}$ under efficiency, we have $\rho\neq 1$ and $\gamma^L\neq\frac{1}{\omega}$ under inefficiency. Furthermore, after rearranging the price–quantity relation in Equation (1.67), the emission response in the tax regime is

$$e(\tau;\theta)=\frac{b_0\alpha_0}{c_0}+\frac{b_1\alpha_1}{c_1}+\left(\frac{c_0\alpha_1+c_1\alpha_0}{c_1c_0}\right)\theta-\frac{\tau}{\gamma^L}. \tag{1.68}$$

We denote the marginal damage curve by md. By assumption, it remains stable.

We illustrate the basic difference between efficient and inefficient implementation in a stripped-down framework of Figure 1.2. In particular, there is only a single realization $\theta=\underline{\theta}$ displayed. We do not derive the policies but rather say that the fixed tax rate τ and fixed quota L represent the policies. Consider first the efficient implementation. We have $mb=mb^0(e;\underline{\theta})$ in Figure 1.2. The taxed emissions e^0 are determined[13] by the intersection $\tau=mb^0(e^0;\underline{\theta})$ while the emissions under quantity implementation stay fixed and are equal to quota L. Had the agency the possibility to reset the policy, it would implement the quantity e that satisfies $mb^0(e;\underline{\theta})=md(e)$; that is, the level implied by the intersection of the *ex-post* marginal curves. As it is unable to do that, certain welfare losses inevitably arise. Specifically, in Figure

[12] We refer to Section 1.3.4.

[13] We comment on the efficient subsidized implementation below.

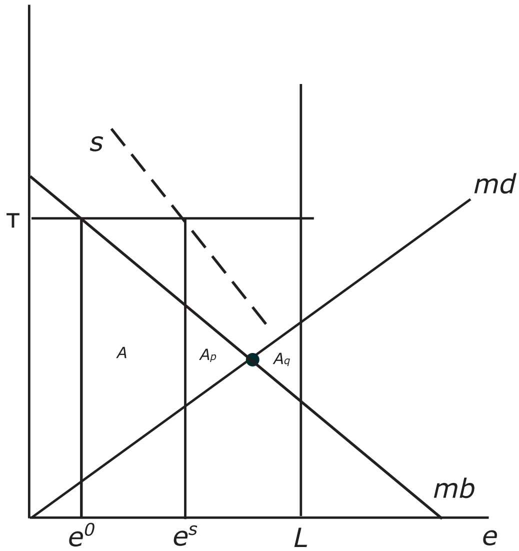

Figure 1.2 Comparative Advantage in Sterilized System: Efficient and Inefficient Implementations Compared

1.2, area A_q is the loss in welfare that the commitment to a fixed level of emissions induces. Similarly, area $A + A_p$ is the loss in welfare that the commitment to a fixed price level induces.

Consider next the inefficient implementation in Figure 1.2. This time, $mb = mb^s(e;\underline{\theta})$ and $mb^s(e;\underline{\theta}) \neq mb^0(e;\underline{\theta})$. Furthermore, the taxed emissions e^s are determined by the intersection between lines τ and $s(e^s;\underline{\theta})$. We illustrated earlier that $s(e;\underline{\theta})$ is everywhere higher than $mb^s(e;\underline{\theta})$. Thus, the realized emissions are strictly lower than the emissions implied by the efficiency condition $\tau = mb^s(e;\underline{\theta})$. In every case, had the agency the possibility to reset the policy, it would implement the quantity e that satisfies $mb^s(e;\underline{\theta}) = md(e)$. Area A_q is the welfare loss that a commitment to a fixed level of emissions induces, while area A_p is the corresponding welfare loss that a commitment to a fixed price level creates. In particular, had the implementation been efficient, the tax welfare loss would have been $A + A_p$. Area A displays the additional tax advantage that inefficient implementation induces in instrument choice.

We emphasize that area A evolves only if both the marginal benefit and the price function differ from the efficient marginal benefit function. For example, if a policy applies efficient but positive thresholds, then this condition is not satisfied. Rather, it holds that $mb^e = mb^e(e;\underline{\theta}) = mb^0$ and $e^e = e^0$. Consequently, the efficiency analysis in Figure 1.2 remains valid.

1.4.6 Prices Versus Quantities with Volume Effect

Next, we assume that the regulatory agency commits to the number of auctioned permits in the quantity policy. This policy does not fix the aggregate level of emissions. This feature is remarkable. We now have an instrument where neither the price nor the quantity remains immune to the realizations of the uncertainty. This property generates a response called a volume effect.

Looking back at the definition in Equation (1.55), we see how the volume effect depends on the different variances of prices and quantities. It is based on the value of the quotient

$$\frac{Var(e(I)) - Var(e(J))}{Var(I) - Var(J)}.$$

In fact, the smaller the variance both in the price and in the quantity, the better it is for instrument s. We may think that the volume effect tries to catch the trade-off between the price and the quantity variances.

The definition of the volume effect (Equation (1.55)) together with variance formulas (Equations (1.59) and (1.60)) imply that

$$\Theta(\tau, p^l(\theta)) = 2\frac{\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}}{R_0(p^l)\left(\frac{\alpha_0}{c_0}\right) + R_1(p^l)\left(\frac{\alpha_1}{c_1}\right)} - 1 \equiv 2q - 1. \tag{1.69}$$

Thus,

$$q = \frac{\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}}{R_0(p^l)\left(\frac{\alpha_0}{c_0}\right) + R_1(p^l)\left(\frac{\alpha_1}{c_1}\right)},$$

and the volume effect does not vanish as long as $q \neq 1$. As for the size of q, we define $\underline{k} = \min(a, 1)$ and $\overline{k} = \max(a, 1)$. Then, we have $q(\underline{k}) = q(\overline{k}) = 1$ and

$$\begin{aligned} q(k) &> 1, 0 < k < \underline{k} \\ 0 < q(k) &< 1, \underline{k} < k < \overline{k} \\ q(k) &> 1, k > \overline{k}. \end{aligned} \tag{1.70}$$

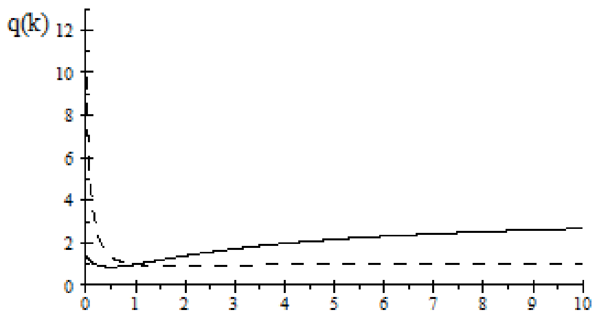

Figure 1.3 Two Graphs of the Function $q(k)$. Parameters are $a = 0.2$ and $s = 2$ (the solid line) and $a = 5$ and $s = 2$ (the dashed line).

To see this pattern, use definitions in Equations (1.47) and (1.48) in Section 1.4.1 and write the value of $q(k)$ as

$$q(k) = \frac{1+uk^2}{1+uak}\frac{1+ua}{1+uk}. \tag{1.71}$$

Specifically, if $q = 1$, then

$$u\left(k^2 + a - k(a+1)\right) = H(k) = 0, \tag{1.72}$$

or, equivalently,

$$u(k-1)(k-a) = 0. \tag{1.73}$$

Inequalities in (1.70) follow, as the function $H(k)$ in (1.72) is seen to be a parabola opening up. We draw two functions of $q(k)$ in Figure 1.3. The applied parameters equal $a = 0.2$ and $u = 2$ (the solid line) and $a = 5$ and $u = 2$ (the dashed line). The critical values of k (where $q = 1$) are $k = 1$ and $k = 0.2$ (the solid line) and $k = 1$ and $k = 5$ (the dashed line).

Our general conclusion is that the decision to abandon a fixed quota in the quantity policy has the potential to yield a volume effect that influences the choice between prices and quantities. More specifically, note first that the values $q < 1$ favor the tax instrument, while the values $q > 1$ favor tradable permits. The influence is seen to disappear at $q = 1$. By Equation (1.73), the volume effect will vanish at critical points $k = a$ and $k = 1$. Remember that these are the critical values in Table 1.2 that induce $e(p^l(\theta)) = e(p^L(\theta))) = \bar{e}$, so the non-sterilized system induces fixed emissions at these critical points. It is then no wonder that the volume effect vanishes at these points.

Second, we study the behavior of q in terms of k above, not in terms of a or u. This choice naturally arises, as we are interested in the changes that subsidization creates. We can also infer from the function $H(k)$ (Equation (1.72)) that the changes in parameters a and u affect the shape of function $q(k)$, but preserve the order of inequalities in (1.70).

Third, the value of q is positive and takes values larger than one at the extremes. The latter of these facts follows, as we have

$$q(0) = 1 + ua \text{ and } \underset{k \longrightarrow \infty}{q(k)} = \frac{\frac{1}{k^2} + u}{\frac{1}{k} + ua} \frac{1 + ua}{\frac{1}{k} + u} \underset{k \longrightarrow \infty}{\longrightarrow} 1 + \frac{1}{ua}.$$

Fourth, with factor q now at our disposal, we write

$$e(p^l(\theta)) = \overline{e} + \left(1 - \frac{1}{q}\right) \frac{c_1\alpha_0 + c_0\alpha_1}{c_1c_0}\theta. \tag{1.74}$$

We call the quantity

$$\frac{c_1\alpha_0 + c_0\alpha_1}{c_1c_0}\theta$$

the tax response. Whenever $q = 1$, the emissions are fixed so the quantity instrument cleans the entire tax response away. Interestingly, if $q > 1$, then $0 < (1 - \frac{1}{q}) < 1$, and we say that the emissions partially adapt. Partial adaptation means that the sectors under the quantity system do not abate the entire response, but only a fraction of it. As we explain above, this particular effect favors the quantity instrument because it relaxes the problematically stringent quota a bit. If $q < 1$ instead, then the system of non-sterilized permits induces perverse behavior. We call the response perverse because a positive productivity shock θ induces a response that will yield $e(p^l(\theta)) < \overline{e}$. In summary, whenever $q > 1$, the volume effect is favorable to the quantities, so the non-sterilized permits induce non-perverse behavior.[14]

[14] We mentioned the possibility that one instrument may become unanimously preferred over another. In the present context, this will occur, if

$$Var\left(e\left(p^l\right)\right) > Var(e(\tau)).$$

As

$$Var(e(\tau)) - Var\left(e\left(p^l\right)\right) = Var(e(\tau))\frac{1}{q}\left(2 - \frac{1}{q}\right),$$

Finally, we briefly note about one specific outcome. It is related to the assumption that $\alpha_1 = \alpha_0 = \alpha$, or equivalently, to the assumption that $a = 1$. Referring to the definition of the volume effect (Equation (1.69)), we may state that

$$\Theta(\tau, p^l(\theta)) \gtreqless 1 \Leftrightarrow q \gtreqless 1.$$

Now, by Equation (1.73),

$$q \gtreqless 1 \Leftrightarrow u(k-1)^2 \gtreqless 0.$$

As far as $u(k-1)^2 > 0$, then $q > 1$, so $\Theta(\tau, p^l(\theta)) > 1$. Consequently, we may state that the volume effect invariably favors quantities.[15]

1.4.7 Prices Versus Quantities: Volume Effect and Cost Effect Combined

Our framework produces a rich variety of cost and volume effects. This means that we cannot give a general prediction of their joint effect at the outset. We say that volume effect dominates if $\frac{\Theta(\tau, p^l(\theta))}{\rho} > 1$. By Equations (1.58), (1.69), and (1.71),

$$\Theta(\tau, p^l(\theta)) - \rho = \left(2\frac{1+uk^2}{1+uak}\frac{1+ua}{1+uk} - 1\right) - \left(\frac{(1+uk^2)(1+ua^2)}{(1+uak)^2}\right),$$

or, after some manipulation (see Appendix A.9),

$$\Theta(\tau, p^l(\theta)) - \rho = \frac{1+uk^2}{1+uka}\frac{u}{1+uk}(a-k)\left(\frac{1-k}{1+uk^2} + \frac{1-a}{1+uak}\right). \tag{1.75}$$

Clearly, the relative size between cost and volume effects is determined by the interaction between factors $(a-k)$, $(1-k)$, and $(1-a)$. We may show that there

then if $q < \frac{1}{2}$, the tax instrument becomes the unanimously preferred instrument. While this remains a feasible state of the world, our calculations merely assess it as an atypically extreme outcome. For example, we may calculate that values of a greater than 34 will induce the condition $q(k) < \frac{1}{2}$. The value $a = 34$ in the two-industry model implies that the larger industry is 34 times larger when measured by the emission content of its production. Moreover, as will become clear soon, if $q(k) < 1$, the regulator invariably uses sterilized permits as the quantity instrument.

[15] In this particular context, equations in Rule (1.24) show how efficiency requires that $l_0 = l_1$. Consequently, thresholds need not be zero but they have to be equal in the efficient policy.

indeed exists various feasible values of a and k that may produce either $\frac{\Theta(\tau,p^l(\theta))}{\rho} > 1$ or $\frac{\Theta(\tau,p^l(\theta))}{\rho} < 1$.

Our calculations imply that the relationship between the cost and volume effects is independent of u but strongly depends on a. Furthermore, in terms of Figure 1.3, we can say (for a given u and a) that $\frac{\Theta(\tau,p^l(\theta))}{\rho} < 1$ over the intermediate values of k such that $\underline{k} < k < \overline{k}$. This may not be a surprise because (by Equation (1.70)) $q < 1$ over the domain $\underline{k} < k < \overline{k}$, so it holds that $\Theta(\tau, p^l(\theta)) < 1$ in there. As far as $\rho > 1$, the result $\frac{\Theta(\tau,p^l(\theta))}{\rho} < 1$ will follow. However, we cannot say that the result $\frac{\Theta(\tau,p^l(\theta))}{\rho} > 1$ holds over the extreme values of k (where $k < \underline{k}$ or $k > \overline{k}$). Rather, we can say that the result $\frac{\Theta(\tau,p^l(\theta))}{\rho} > 1$ holds over the region $k < \underline{k}$ or over the region $k > \overline{k}$. The alternative that eventually results will depend on the value of a.

1.4.8 Quantities Versus Quantities

Next, we generalize the discussion by allowing the regulator the freedom to choose between the sterilized and the non-sterilized systems. This means a choice between two quantity instruments. This analysis is novel. Previous studies have not explicitly come across similar comparisons. Furthermore, the result has repercussions as well, since it affects the (more traditional) choice between the prices and the quantities (see next section).

We modify the measure in Equation (1.57) to yield

$$\Delta(p^l(\theta), p^L(\theta)) = Var(p^L(\theta))\frac{1-v}{2}\frac{\rho}{(\gamma^L)^2}\left(\gamma - \frac{\Theta}{\rho}d\right), \tag{1.76}$$

where it holds that $\Theta \equiv \Theta(p^l(\theta), p^L(\theta))$ and $v \equiv v(p^l(\theta), p^L(\theta))$. After some tedious but straightforward calculations,[16] we have

$$v(p^l(\theta), p^L(\theta)) = \frac{Var(p^l(\theta))}{Var(p^L(\theta))} = \left(\frac{1}{q}\right)^2 \tag{1.77}$$

[16]The calculations are in Appendix A.10. The value of $\Theta(p^l(\theta), p^L(\theta))$ is calculated in Part I, while $v(p^l(\theta), p^L(\theta))$ is calculated in Part II.

and

$$\Theta(p^l(\theta), p^L(\theta)) = \frac{q-1}{q+1}. \tag{1.78}$$

Interestingly, the non-sterilized system does not always provide a lower price variance than the sterilized system, but it does so whenever $q > 1$. We say that non-sterilized permits induce non-perverse behavior over this area.[17]

The measure $\Theta(p^l(\theta), p^L(\theta))$ differs from an earlier counterpart ($\Theta(\tau, p^l(\theta))$ in Equation (1.69)), but it shares dependency on factor q. A striking difference concerns factor $v = v(p^l(\theta), p^L(\theta))$, as it now takes values strictly greater than zero. However, the size of factor v does not affect the choice of the instrument. To see this, consider the size of Δ under different values of q. First, if $q = 1$, then $v = 1$ and $\Delta = 0$. Both systems produce a similar variance in emissions and in prices at the critical points $k = a$ and $k = 1$ (see Table 1.2), so the regulator is indifferent between the instruments. Second, if $q < 1$, then $\Theta < 0$ and $v > 1$, and it unanimously holds that $\Delta < 0$. In other words, the values $q < 1$ invariably imply a unanimous advantage for the sterilized system. This result does not come as a surprise, as we have already indicated that the values $q < 1$ induce perverse behavior in the non-sterilized system. Finally, if $q > 1$, then $0 < v < 1$. The value of v does mitigate the magnitude of Δ, but it does not affect the choice between the instruments.

Whenever $q > 1$, we have $0 < \Theta(p^l(\theta), p^L(\theta)) < 1$. Thus, over the non-perverse region, the volume effect works for the non-sterilized system. Recall our earlier discussion that the cost effect favors the instrument with the lower variance in price. Here, over the non-perverse region ($q > 1$), it holds (by Equation (1.77)) that

$$Var(p^L(\theta) > Var(p^l(\theta)).$$

With the help of representation in Equation (1.76), the reader can convince herself that the cost effect indeed favors the non-sterilized permits. In summary, whenever $q > 1$, we have $\frac{\Theta}{\rho} < 1$, so the combined cost-volume effect is invariably favorable to the non-sterilized permit system.

[17]We refer back to the definition of the volume effect in Equation (2.68). In the present context, we have $Var(e(J)) = Var(e(p^L)) = 0$. So, whenever $\Theta(p^l(\theta), p^L(\theta)) > 0$, we must have $Var(p^l) < Var(p^L)$.

1.5 Instrument Choice under Uncertainty

1.5.1 Comparison of Three Instruments

So far, we have qualified two effects that the subsidization policy potentially triggers: the cost effect and the volume effect. In our final section, we incorporate these effects into a full-scale study of the comparative advantage. We start by collecting the various formulas in the first column of Table 1.3.[18] The second column displays the corresponding indifferences between two instruments. The column is based on the fact that a regulator is indifferent between instruments I and J if $\Delta(I,J)=0$. By Equation (1.57), the indifference is satisfied whenever

$$m=\frac{\rho}{\Theta},$$

where $m \equiv \frac{d}{\gamma}$. Remember that factor ρ is the cost effect and $\Theta \equiv \Theta(I,J)$ is the volume effect. We have $\rho \geq 1$ (Equation (1.58)), while various volume effects are given by Equations (1.64), (1.69), and (1.78).

Table 1.3 Prices vs. Quantities

$\Delta(\tau, p^L(\theta))=\rho\frac{Var(p^0(\theta))}{2\gamma^2}\left(\gamma-\frac{\Theta(\tau,p^L(\theta))}{\rho}d\right)$	$m=f_{\tau L}(q;\rho)=\rho$
$\Delta(\tau, p^l(\theta))=\rho v_{\tau l}\frac{Var(p^0(\theta))}{2\gamma^2}\left(\gamma-\frac{\Theta(\tau,p^l(\theta))}{\rho}d\right)$	$m=f_{\tau l}(q;\rho)=\frac{1}{2q-1}\rho$
$\Delta(p^l(\theta), p^L(\theta))=\rho v_{lL}\frac{Var(p^0(\theta))}{2\gamma^2}\left(\gamma-\frac{\Theta(p^l(\theta),p^L(\theta))}{\rho}d\right)$	$m=f_{lL}(q;\rho)=\frac{q+1}{q-1}\rho$

We illustrate the choice between the subsidized prices and quantities with the help of Figure 1.4. The figure is drawn in the m,q–plane. Part (a) consists of different areas and different borders between these areas. Points inside the borders represent a strict preference for one instrument over the other two, while a point along the border implies indifference between two instruments. As for the shape of the different areas, we have

[18] We have unified the notation by incorporating the common variance into the various formulas. This unification generates two multiplicative factors: $v_{\tau l}$ and v_{lL}. However, based on the earlier analysis, it should be clear that these factors are strictly positive in the comparisons to come.

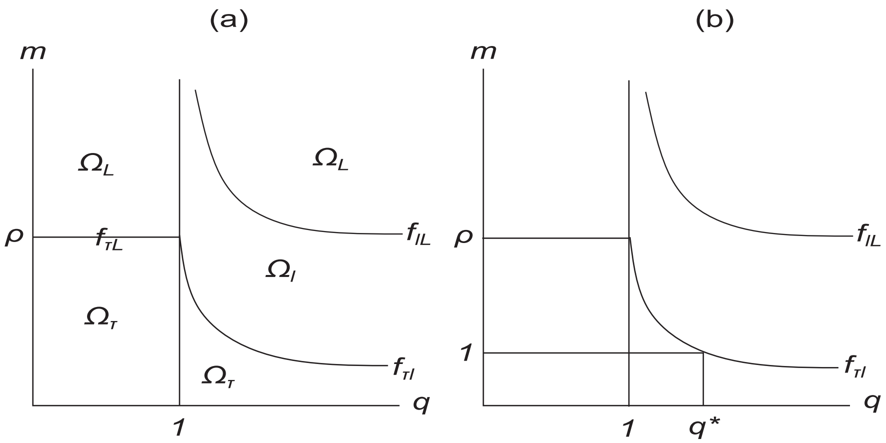

Figure 1.4 The Instrument Choice under Uncertainty (a); The Critical Value q^* (b)

$$\frac{df_{\tau l}(q;\rho)}{dq} = -\frac{2\rho}{(2q-1)^2} < 0, f_{\tau l}(q;\rho) \underset{q \longrightarrow 1+}{\longrightarrow} \rho,, f_{\tau l}(q;\rho) \underset{q \longrightarrow \infty}{\longrightarrow} 0$$

and

$$\frac{df_{lL}(q;\rho)}{dq} = -\frac{2\rho}{(q-1)^2} < 0, f_{lL}(q;\rho) \underset{q \longrightarrow 1+}{\longrightarrow} \infty, f_{lL}(q;\rho) \underset{q \longrightarrow \infty}{\longrightarrow} \rho.$$

Figure 1.4(a) displays these properties.

Figure 1.4(a) also shows how the changes in q will affect the instrument choice. Especially note how we draw the figure for a given level of the cost effect. Furthermore, we will have a non-degenerate figure (like Figure 1.4(a)), as long as we study inefficient outcomes. That is, we must have $k \neq a$. If $k = a$, then efficiency prevails, and the figure will collapse into a single vertical line at $q = 1$. We are then back in a traditional analysis (Equation (1.63)). We have $\rho = 1$, so the quantity (the price) is the choice as long as $m > 1$ ($m < 1$).

In the world of inefficiency, the size of q strongly affects the choice of the instruments. Specifically, it is important whether q is smaller, greater, or equal to one. First, consider the case in which $q < 1$. Based on the discussion above, we know that the regulator always sterilizes in the permit regime. Then, when it comes to the choice between sterilized permits and tax, the modified Weitzman rule $\Delta(\tau, p^L(\theta))$ should be conducted. In Figure 1.4(a), the sterilized permits are chosen if the point lies in area Ω_L, while a point in area Ω_τ implies that the tax is chosen instead. The border between these two areas is a set of indifferences, so the points at the border satisfy $m = \rho > 1$.

Second, at $q = 1$, the regulator is indifferent between the sterilized and the non-sterilized permits. As for the choice between prices and quantities, the indifference occurs again at $m = \rho$. If $m < \rho$, then the price is chosen, and if $m > \rho$, then the quantity should be chosen. Note carefully that we must have $k = 1$ so that both $q = 1$ and $\rho > 1$ will hold.

Third, we may have $q > 1$. For a given q in this area, we have $m_{lL} = f_{lL}(q;\rho) > m_{\tau l} = f_{\tau l}(q;\rho)$. Thus, the tax is the preferred choice over $0 \leq m < m_{\tau l}$, the non-sterilized permits are preferred over $m_{\tau l} < m < m_{lL}$, and the non-sterilized permits are preferred over $m > m_{lL}$. In Figure 1.4(a), for every given (finite) value of $q > 1$, there exists three non-empty sets for which the tax, the sterilized permits, and the non-sterilized permits are the preferred choices, respectively. Then, in addition to the areas Ω_L and Ω_τ, we also have an area Ω_l, where the system of non-sterilized permits is the preferred choice.

We then state:

Proposition 1 *Assume that the regulator is not forced to keep the permit endowment at a fixed level but it may adapt it to the entries and exits of the polluting firms. In this case, two factors cover the influence of subsidization on comparative advantage. Factor $\rho \geq 1$ records a pure inefficiency effect that the subsidization of the polluting sectors causes. Factor $q > 0$ determines the level of the volume effect, which is a result of the non-fixed quota. If we denote the relation between the slopes of the marginal damage and benefit functions $\frac{d}{\gamma}$ by m, we have the following result:*

i. If $q < 1$, then only taxes and sterilized permits are used. Moreover, whenever $m \lesseqgtr \rho$, then $\Delta(\tau, p^L(\theta)) \gtreqless 0$.

ii. If $q > 1$, then taxes and both sterilized and non-sterilized permits are applied. If $m < \rho$, then only taxes and non-sterilized permits are used. If $m > \rho$, then only sterilized and non-sterilized permits are used. (The case $m = \rho$ applies only asymptotically.) The indifferences $\Delta(\tau, p^l(\theta)) = 0$ and $\Delta(p^l(\theta), p^L(\theta)) = 0$ are given by decreasing functions $m = f_{\tau l}(q;\rho)$ and $m = f_{lL}(q;\rho)$, respectively.

iii. If $q = 1$, then the regulator is indifferent between the sterilized permits and the non-sterilized permits. Whenever $m \lesseqgtr \rho$, then $\Delta(\tau, p^L(\theta)) = \Delta(\tau, p^l(\theta)) \gtreqless 0$. However, depending on the efficiency of the allocation, it either holds that $\rho = 1$ or $\rho > 1$.

The proof follows our earlier treatment. Furthermore, we state

Proposition 2 *If $q < 1$, then the advantage of taxes over quantities invariably increases as compared to the case with efficient emission allocation. If $q > 1$, then there exists a $q = q^*$, so that if $q < q^*$, then the advantage of taxes over quantities increases; if $q > q^*$, then the advantage of quantities over taxes increases in comparison to the case with efficient emission allocation.*

Proof. We have drawn an auxiliary line at $m = 1$ in part (b) of Figure 1.4. This line determines the choice between prices and quantities in an efficient setting. In such a setting, if $m \lesseqgtr 1$, then $\Delta(\tau, p^0(\theta)) \gtreqless 0$ (see Equation (1.63)). Next, choose an arbitrary $q < 1$. We now know with certainty that the comparative advantage of prices has increased. This follows, as there now exists values of m that satisfy $m > 1$, but the tax is the preferred instrument. Instead, if we choose an arbitrary $q > 1$, the conclusion about the influence depends on the specific value that q takes. In general, with low values of q, the comparative advantage of prices again increases. This happens as long as $q < q^*$, where q^* is determined by the condition $f_{\tau l}(q^*; \rho) = 1$. Thus, over the region $q < q^*$, there exist values of m that satisfy $m > 1$, but the tax is the preferred instrument. Finally, if $q > q^*$, the effect of the endogenous permit supply is strong enough so that the comparative advantage of the quantities increases. ■

Regarding the result in Proposition 2, Table 1.3 shows us that quantities gain an additional advantage as long as $\Theta(\tau, p^l(\theta)) > \rho$. In other words, the influence of uncertainty on instrument choice changes at the precise moment when the volume effect exceeds the cost effect.

We want to link our results to an earlier paper by Montero ([46], [49]). We already mentioned in the Introduction that the regulator in his model shares the nature of our regulator as she loses her full authority in a discrete framework. In Montero [49], there is a governmental incapability to implement the regulation without cheating. Interestingly, the incapability means endogenous emissions in the system of tradable permits. The aggregate permit endowment as such is truly fixed, but because of the cheating, the level of emissions exceeds the level of the permit holdings in some firms. Therefore, the aggregate level of realized emissions exceeds the aggregate cap. Montero finds that the endogeneity invariably favors the quantity instrument in the instrument choice. Montero's insight is then that the cheating induces the quantity instrument beneficially away from the stringent quota.[19]

[19] For this interpretation, a useful reference is the hybrid instrument first introduced by Roberts and

The apparent similarity between our model and the Montero [49] model concerns the endogeneity of tradable emissions. However, endogenous emissions in permit trading are not inevitable in our model as the regulator can decide whether to sterilize the permit fluctuations (and the subsequent emission fluctuations). Moreover, the non-fixed emissions do not necessarily favor the quantity instrument in our model as it does in Montero. Finally, while Montero's model operates under inefficiencies as well, it does not stress the distinction between the cost and the volume effect. Regarding this last point, we emphasize that the volume effect requires inefficiencies to operate; that is, the volume effect arises only in the market, where the firms trade permits under inefficient trading ratios.

Finally, we briefly discuss the industry where $\alpha_1 = \alpha_0 = \alpha$. We calculate above that $q > 1$ in this particular context. Furthermore, we write Equation (1.75) now as

$$\Theta(\tau, p^l(\theta)) - \rho = u\frac{1+uk^2}{(1+uk)^2}\frac{(1-k)^2}{1+uk^2} > 0. \tag{1.79}$$

Thus, in terms of Figure 1.4, we have $q^* = 1$, so we do not have an area where $q < 1$. In this type of polluting industry, the volume effect invariably dominates the cost effect, so the relative position of the quantity instrument is improved.

1.5.2 Extension of the Basic Model

We derive a basic model above to understand better the consequences that inefficient substitution causes in a regulatory framework with discrete choices. In explaining our model in the beginning of the analysis, we also discussed that we somewhat restrict the generality as we study only permit demanders in both polluting sectors. Here, we expand the framework as we briefly discuss a model that allows both permit surpluses and deficits in the polluting industry.

Our extension is based on the idea that firms may switch between different types of technologies. Recall that we discussed in the Introduction a polluting sector (see Figure (1)) where firms are divided into subgroups. This division can be explained by the different technology modifications that the firms apply. We may think of an old brown technology that has been applied well before the environmental regulation. The technology is still working but it is becoming expensive because of regulation.

Spence [62].

However, firms may modify the brown technology to be greener by switching to less polluting inputs or by installing end-of-pipe purification technologies.

Assume that firms will differ in their possibilities in modifying the brown technology. We refer to Section 0.2.2 in the Introduction as we write benefits for unit λ that uses technology j within sector i as

$$B_i^j(\lambda) = b_i^j + \theta_i^j - c_i^j \lambda,$$

where $i = 0, 1$ and $j = b, g$. Thus, if $j = b$, the unit applies the old brown technology, while $j = g$ implies that the unit has modified the technology to be greener. Furthermore, to keep in line with this chapter, we assume that the policy applies sector-specific subsidies. We then have $l_i^b = l_i^g = l_i$, so the profit function becomes

$$\Pi_i^j(\lambda) = B_i^j(\lambda) - s(\alpha_i^j - l_i),$$

where $i = 0, 1$ and $j = b, g$.

We discuss in Section 0.2.2 that there exist two cut-off units within the industries. First, unit λ_i^b satisfies

$$\Pi_i^b(\lambda_i^b) = 0$$

in industry i. The unit is indifferent between producing and exiting the market. Second, there is unit λ_i^g in industry i that satisfies

$$\Pi_i^g(\lambda_i^g) = \Pi_i^b(\lambda_i^g).$$

In the present context, this unit is indifferent between modifying the technology and using the old technology without modifications. Solving the cut-off units gives us

$$\lambda_i^g = \frac{\Delta b_i + \Delta\theta_i - s\Delta\alpha_i}{\Delta c_i}$$

and

$$\lambda_i^b = \frac{b_i^b + \theta_i^b - s(\alpha_i^b - l_i)}{c_i^b},$$

where $\Delta b_i = b_i^g - b_i^b > 0$, $\Delta\theta_i = \theta_i^g - \theta_i^b$, $\Delta\alpha_i = \alpha_i^g - \alpha_i^b < 0$, and $i = 0, 1$. Unit λ_i^g does not depend on the value of l_i, so the subsidization does not affect the determination of it. This particular outcome follows as subsidies are paid to producing units but they are not conditioned on the applied technology.

One may continue the analysis towards the determination of aggregate variables and eventually towards the instrument choice. We consider here only the equilibrium permit price under a non-sterilized system. By Equation (1.15), the equilibrium satisfies

$$l=\int_0^{\lambda_0^g}(\alpha_0^g-l_0)d\lambda+\int_{\lambda_0^g}^{\lambda_0^b}(\alpha_0^b-l_0)d\lambda+\int_0^{\lambda_1^g}(\alpha_1^g-l_1)d\lambda+\int_{\lambda_1^g}^{\lambda_1^b}(\alpha_1^b-l_1)d\lambda,$$

where l is the number of auctioned permits. The first part in the right-hand side corresponds to net demand in sector zero, while the second part is the sector one net demand. Note, in particular, that this new framework allows the presence of supramarginal thresholds ($\alpha_i^g-l_i<0$) as well. Consequently, green firms may end up selling their excess permits in the permit markets. We may write the equilibrium condition above in the alternative form

$$l=\lambda_0^g\Delta\alpha_0+\lambda_0^b(\alpha_0^b-l_0)+\lambda_1^g\Delta\alpha_1+\lambda_1^b(\alpha_1^b-l_1),$$

so that after inserting the various cut-off units into the condition, the equilibrium price is

$$\begin{aligned}p^l=&\frac{1}{\gamma^l}\left(\frac{(\Delta b_0-\Delta\theta_0)}{\Delta c_0}\Delta\alpha_0+\frac{\Delta b_1-\Delta\theta_1}{\Delta c_1}\Delta\alpha_1\right)\\&+\frac{1}{\gamma^l}\left(\frac{(b_1^b-\theta_1^b)}{c_0^b}(\alpha_0^b-l_0)+\frac{b_1^b-\theta_1^b}{c_1^b}(\alpha_1^b-l_1)\right)-\frac{l}{\gamma^l},\end{aligned}$$

where

$$\gamma^l=\frac{\Delta c_1\Delta c_0c_0^bc_1^b}{c_0^bc_1^b\left(\Delta c_1\Delta\alpha_0^2+\Delta c_0\Delta\alpha_1^2\right)+\Delta c_1\Delta c_0\left(c_1^b(\alpha_0^b-l_0)^2+c_0^b(\alpha_1^b-l_1)^2\right)}.$$

The non-sterilized price p^l is comparable to another non-sterilized price, namely to the price in Equation (1.16). Note that these prices differ in the absence of subsidization ($l_0=l_1=0$) as well. They differ because the underlying production technologies are different. Note also that the subsidization (either $l_0>0$ or $l_1>0$ or both) enters the non-sterilized permit here in a similar fashion as it does earlier (see

Equation (1.16)).

1.6 Concluding Remarks

The original Weitzman [84] model provides a fundamental instrument choice rule by assuming an efficient allocation of emissions among the regulated units. We extend this framework by incorporating inefficient subsidization into regulation. Specifically, our model describes subsidization within regulation where the number of regulated units fluctuates. This creates a new challenge to environmental agency, as the quantity instrument does not automatically fix the level of emissions. Rather, the agency has to choose whether it should fix the level of emissions. In any case, environmental tax remains an option in regulation, and we end up studying subsidized instrument choices between prices and quantities. We show that the fluctuating number of firms remains an issue in the instrument choice as long as subsidization remains inefficient.

Overall, inefficient subsidization in our model produces two effects that together may favor either prices or quantities in the instrument choice. The cost effect records the effect that the given amount of emission is inefficiently distributed in the polluting industry. This effect invariably favors the price instrument. The volume effect records the effect that tradable permits apply a non-fixed quota. The volume effect is interesting because the Weitzman model [84] demonstrates how fixed emissions is a major difficulty in quantity regulation. Our analysis shows that a non-fixed quota is not necessarily beneficial to the quantity instrument. However, there certainly exist regimes where a non-fixed quota is beneficial and the advantages may be so great that the inefficient subsidization favors the quantity instrument.

We assume that only discrete projects reduce aggregate emissions. In our main analysis, the closure of a polluting unit is the only means to reduce emissions, so these choices dominate the policy content. One implication of this assumption is that every active polluting unit is a permit buyer in the permit market. This setting is simple and provides us the basic intuition about the inefficient substitution. In the future, we should study settings that allow for both permit surpluses and deficits in the market. Towards that end, we discuss an augmented model at the end of the chapter. This model is based on the assumption that some polluters find technology updating profitable. We continue with the topic when we concentrate on investments, investment subsidies, and reciprocal trade in Chapter 2.

Our model emphasizes the role of auctions in the implementation of tradable permits. Overall, we think that the presence of an additional permit reserve (distributed in permit auctions) will provide stability in the permit markets. It also constitutes a tool in permit policy. Our focus on auctions leaves the method of grandfathering (free allocations) in the background. Grandfathering can be incorporated easily into the framework: just set the number of auctioned permits to zero in the system of non-sterilized permits. We have a few comments on this approach. First, as grandfathering is just a special case of subsidization in our framework, our findings of subsidization apply to it also. Most notably, the emission allocation may become inefficient. Second, the system of sterilized permits becomes infeasible under grandfathering. If the environmental agency loses the auctions, it cannot sustain a fixed level of emissions in the policy. We also like to note that policies do not only regulate existing units but future units as well. As we discussed in the Introduction, the existing permit policies hold a reserve of permits to be auctioned to new units.

Our environmental agency is a restricted regulator. This is because the agency takes some policy variables exogenously given. One may argue that the agency can in practice be even more restricted than ours. After all, the agency is able to determine the overall policy strictness, as it is free to choose the imputed expected unit price of emissions in the regulation. We agree that this comment is relevant. Thinking politically, as we do in this chapter, it is not only the size of the payments but also the scope of the aggregate policy that matter among the regulated units. If the agency is not only restricted by the subsidization but also by policy strictness, we may argue that the analysis about the instrument choices above remains valid. The basic reason is that we have assumed that marginal benefit and damage functions are linear. Consequently, it does not matter whether we study instrument choice under optimized or non-optimized policy, as long as both instruments implement the same unit price of emissions and that marginal functions are linear in the neighborhood of the unit price.

Finally, note that the subsidy $S_i(p)$ depends on the permit price. Consider for a moment an alternative permit system in which the subsidy is fixed and equals

$$S_i(\tau) = \tau l_i, \tag{1.80}$$

τ is the (fixed) unit price, and l_i is an emission threshold. We have a kind of hybrid permit system. While it does fix the subsidy payment, it does not fix the unit price of emissions. If both prices and quantities apply the same fixed subsidy, we can show that the comparison between instruments reverts to the original Weitzman rule. To yield intuition in these cases, we rewrite the number of firms in sector i (Equations (1.4) as

$$\lambda_i^* = \frac{\widetilde{b}_i + \theta_i - s\alpha_i}{c_i},$$

where

$$\widetilde{b}_i = b_i + \tau l_i,$$

s is a unit price of emissions, and $i = 0, 1$. Consequently, $\widetilde{b}_i$ is effectively a new constant of the marginal benefit function. Our analysis in the main text shows how changes in the constant affect policy strictness but do not affect the instrument choice. We note that the same logic will apply with any fixed subsidy $S_i(s) = F_i$ that is paid both in quantity and price regimes. It affects overall policy strictness but does not affect the instrument choice.

2 IMPERFECT PARTICIPATION

2.1 Some Background

Tradable permits and environmental taxes are the key means or "instruments" for implementing an environmental policy. In the literature, these two instruments are referred to as market-based instruments, and their use is particularly powerful when the number of regulated firms is large (see Stavins [74]). However, in some of the actual policies, only a subset of the polluters is being regulated. This is apparent in the sulfur dioxide emissions trading program in the United States (Ellerman [13]), in the European Union Emissions Trading System (European Commission [17]), or more generally, in the various flexibility mechanisms applied for greenhouse gas reduction (Newell, Pizer, & Raimi [52]). This raises questions about the reasons behind partial participation. Admittedly, the less-than-full participation may not always represent regulatory failure, rather it reflects regulatory judgment. For example, in learning-by-doing, the gradual progress from easily regulated companies towards more laborious cases may well yield savings in the implementation costs.

In our theoretical model of this chapter, less-than-full participation in a market-based environmental program is a starting point, but we do not find it socially desirable. We assume that issues of implementation and monitoring of the program do not justify less-than-full participation. Rather, some institutional and political factors explain the scope of regulation. We think that in situations like these, voluntary participation is a potential mechanism to increase the participation rate toward fuller participation. With voluntary participation, the general idea is to offer a carrot to a firm to participate. Specifically, with tradable permits, the program offers a free permit handout to a firm that agrees to cover its emissions through licenses. With environmental taxes, the agency offers a tax exemption to firms that voluntarily pay the Pigouvian tax. Whether permit handouts or tax exemption are applied, they should be attractive enough so that a firm voluntarily participates. Overall, we have two specific questions in mind. First, under what circumstances does voluntary participation raise social welfare? Second, if voluntary participation indeed represents sensible policy, does it affect the choice between market-based instruments, namely, between tradable permits and pollution taxes?

The model used in this paper is valid if the polluting industry consists of several polluters and they emit homogenous pollution. For example, in global warming, numerous polluters emit the same pollution, so it meets the criteria of our model. In fact, the topic of the implementation of carbon reductions has sparked considerable debate (Keohane [31]). While the majority of participants in this debate admit that there is a need for a global carbon price, the decision between using carbon tax or tradable permits for this purpose is still under consideration. Our work thus aims to supplement this discussion, not least because the global scope of the regulation is far from perfect. Furthermore, with regard to the question of voluntary participation, the interest has so far largely focused on expanding the system of tradable permits.[1] We offer a broader view, as we treat pollution tax as an equal alternative in voluntary participation.[2]

Our model is a modified version of the Montero [47] model. Montero draws experience from the sulfur dioxide emissions trading program, which is an example of a phase-in emissions trading program. Most interestingly, the program includes the so-called substitution provision, which allows producers unaffected in the first phase of the program to participate voluntarily. The first phase of the program mandates only a subset of the producers to participate, while the number of mandated units increases in the second phase. Voluntary participants receive an initial allocation of tradable permits and the status of an affected firm. Montero emphasizes the issues of imperfect information and distributional concerns as well as the cost and benefit uncertainty in market design. More generally, distributional concerns bring us to the political economy of policy-making (Farrow [18]). Pollution permits, like environmental taxes, circulate revenues and create winners and losers. In particular, the revenue generated by the initial allocation of permits chiefly determines the scope of voluntary participation. Ultimately, as revenue generating possibilities become limited, Montero [47] shows that the market design turns into a second-best design,

[1] Bento, Kanbur and Leard [5] provide a recent contribution.

[2] If we look at the current global carbon policy implementation, a single global carbon price has not yet emerged. As discussed in Newell *et al.* [52] and in Carl and Fedor [6], the current situation includes the simultaneous use of many different and separate carbon markets, national carbon tax systems, governmental green subsidies, and various development projects.

where voluntary participation becomes inefficient because not every low-cost, non-affected firm opts in at the regulatory optimum.

Our model depicts a phase-in emissions policy with a provision for voluntary participation. Our model follows Montero [47], as distributional concerns make voluntary participation inefficient and cost uncertainty drives our central results. However, the models differ in several respects. First, and most importantly, our focus is not on the design of a phase-in emissions trading program but on the comparison between tradable permits and environmental taxes. We examine how these two different regulatory instruments perform in the implementation of a voluntary participation provision. Second, as the objectives differ so much, the cost uncertainty manifests itself in different ways. Montero asks how the uncertainties affect the permit allocation rules, whereas our work presents a comparative study of instrument choice under uncertainty (Weitzman [84]). Third, our model depicts how investment possibilities in green environmental technology may enhance voluntary participation. Our model does not explain why one of the sectors lies outside the cap, but it gives a clear signal and rationale for enclosing some firms inside the cap on a voluntary basis. Our model shows that voluntary participation provision subsidizes the switch from old brown technology to new green technology.

Generally speaking, the voluntary participation provision benefits environmental policy as a larger part of existing pollution becomes controlled. When it comes to the implementation of these benefits, the distributional properties between the instruments can be unified. In our model, any expected net benefits that a phase-in emission trading program generates can be generated through a phase-in environmental taxation program. However, our main result states that inefficient voluntary participation strongly affects the instrument choice. We arrive at this conclusion by following the framework of Chapter 1, where the traditional Weitzman model is extended to better understand the consequences of the imperfections in the market.[3] Consequently, we regard our current work as an application of this general framework. According to the central story of our model, the implementation of voluntary participation necessarily requires subsidies, and the presence of the subsidies immediately leads to potential inefficiencies, which affect the instrument choice in a fundamental manner.

We summarize our results into three concepts: the scope effect, the cost effect,

[3]Second-best implementation is inherently present in this chapter as it is in Montero [47].

and the volume effect. The scope effect is a novel feature, arising solely due to imperfect participation. The scope effect depicts how the instrument choice is affected when the scope of regulation expands from imperfect towards perfect participation. We show that the scope effect will benefit the quantity instrument. The cost effect depicts the influence of inefficiency on the instrument choice. It records the fact that the scope is increased by applying inefficient subsides. We show that the cost effect favors stable prices. Thus, a tax system with a fixed price has an unambiguous advantage. The volume effect records influences when the quantity rule does not fix the level of emissions. Like in Chapter 1, the basic framework here includes fluctuations in the number of polluting units. However, voluntary participation adds variability in this section as the participation rate becomes a stochastic variable as well. In particular, if the participation rate is strictly lower than one, the agency faces a situation in which it cannot stabilize the level of emissions by fixing the number of permits. This means that that the volume effect becomes an inevitable effect in the inefficient instrument choice.

A recent contribution by Meunier [43] considers instrument choice in a setting where participation can be interpreted as imperfect. It studies a setting in which an unregulated good pollutes alongside a regulated polluting good. However, the approach differs in many ways from ours. Most importantly, firms' discrete decision making is missing in Meunier [43]. This implies, for example, that his framework omits the voluntary participation provision that is central in our work. Furthermore, our model has only one unpriced externality, whereas in Meunier [43] there are more than one. Note that we will consider positive technological externalities as a separate case in Chapter 3.

When studying instrument choice under uncertainty and imperfect participation, our work mixes three distinct branches of the literature: the long-run efficiency properties of taxes and tradable permits (Farrow [18]; Pezzey [54]), voluntary participation (Montero [47]), and inefficient policy implementation under uncertainty (Chapter 1; Montero [49]). In doing so, our work supports the line originating from Weitzman [84]. Recent contributions that share our topics include those of D'Amatoa and Dijkstra [10], Krysiak [34], Shinkuma and Sugeta [68], Krysiak and Oberauner [33], and Mandell [40]. Krysiak and Oberauner [33] and Mandell [40] study the design of a hybrid instrument (Roberts & Spence [62]). The hybrid nature of these policies is indirectly present in our study, as the endogeneity of permit supply has the potential to soften the extreme nature of the quantity instrument. D'Amato and Dijkstra [10], and Krysiak [34] study technical change toward green investments, while Shinkuma and Sugeta [68] incorporate distributional concerns and sequential decision-making into their analysis. Common to all of these studies is the assumption that participation is perfect.

2.2 Regulation without Voluntary Participation

2.2.1 The Industry

This chapter presents studies of three policies that will give rise to three different market structures. We will study markets with perfect participation, zero voluntary participation, and positive voluntary participation. In perfect participation, regulation mandates that every unit in the polluting industry participate. As far as we have two sectors, A and N, in the polluting industry, the policy regulates both sectors. With zero voluntary participation, there is no voluntary program in progress. In our framework, the policy regulates only sector A while sector N stays completely out of it. With positive voluntary participation, non-affected firms become regulated voluntarily. The policy regulates every unit in sector A, while the policy may regulate units in sector N. To draw a difference between cases of perfect participation and positive voluntary participation, we denote sector N by NA under positive voluntary participation. In summary, sectors A and N go along with the whole chapter while the policy will change along the way.

The polluting industry consists of two distinct sectors. The sectors, denoted by A and N, are heterogeneous in terms of emissions-generating technologies but they emit homogenous pollution as a by-product of their production. The benefit for unit η of producing one commodity unit in sector A is

$$B_A(\eta) = b_A + \theta_A - c_A\eta, \tag{2.1}$$

where b_A and c_A are positive constants while θ_A is a random variable. The firms in sector N may choose between two production technologies to produce the commodity unit. The benefit for unit λ after choosing technology i is

$$B_i(\lambda) = b_i + \theta_i - c_i\lambda, \tag{2.2}$$

where $i = 0, 1$. Again, b_i and c_i are positive constants while θ_i is a technology-specific random shock. We assume that $\Delta b \equiv b_1 - b_0 > 0$ and $\Delta c \equiv c_1 - c_0 > 0$. Furthermore, we will denote $\Delta\theta \equiv \theta_0 - \theta_1$.

We assume that uncertainty contains both an aggregate and an idiosyncratic part. The total shock then consists of a technology-specific shock (ϵ_j) and an economy-wide shock (ϵ). Every industry in the economy shares the economy-wide shock. The uncertainty takes an additive form, which means that

$$\theta_j = \frac{\epsilon + \epsilon_j}{2},$$

where $j = 0, 1, A$. In particular,

$$\Delta\theta = \frac{\epsilon_0 - \epsilon_1}{2},$$

so the variable $\Delta\theta$ does not include the aggregate shock. We further assume that the variables ϵ and ϵ_j are identically and independently distributed random variables, so $E(\epsilon) = E(\epsilon_j) = 0$ and $Var(\epsilon) = Var(\epsilon_j) = \sigma^2$, $j = 0, 1, A$. Then, $E(\theta_A) = E(\Delta\theta) = 0$ and $Var(\theta_A) = Var(\Delta\theta) = \sigma^2$.

We denote the unit price of emissions by s. It is part of the environmental payment function $P_j(s, \alpha_j, l_j)$. Specifically, we have

$$P_j(s; \alpha_j, l_j) = s(\alpha_j - l_j),$$

where $j = 0, 1, A$. Parameter α_j is the level of emissions that a firm produces if it utilizes technology j, while l_j is a technology-specific threshold level set by the regulator. Then, the profit for unit η is

$$\Pi_A(\eta) = B_A(\eta) - P_A(s; \alpha_A, l_A) = B_A(\eta) - s(\alpha_A - l_A),$$

while the profit for unit λ using technology i is

$$\Pi_i(\lambda) = B_i(\lambda) - P_i(s; \alpha_i, l_i) = B_i(\lambda) - s(\alpha_i - l_i), \tag{2.3}$$

where $i = 0, 1$. Every parameter in $P_j(s; \alpha_j, l_j)$ is a non-negative figure. Thus, whenever $P_j > 0$, then $l_j < \alpha_j$, and P_j is effectively a fee. Conversely, if $P_j < 0$, then, $l_j > \alpha_j$, and P_j is a compensation[4] to a unit that utilizes technology j, where $j = 0, 1, A$.

Within the sectors, there exist cut-off units that are indifferent between two options. Specifically, the cut-off unit η_A^0 has $B(\eta_A^0) = 0$, so

$$\eta_A^0 = \frac{b_A + \theta_A}{c_A}. \tag{2.4}$$

The cut-off unit η_A^0 emerges if emissions are not regulated at all in sector A. Conversely, the cut-off unit η_A has

$$\Pi_A(\eta_A) = 0, \tag{2.5}$$

so

$$\eta_A = \frac{b_A + \theta_A - s(\alpha_A - l_A)}{c_A}. \tag{2.6}$$

Clearly, if $l_A < \alpha_A$, then we have $\eta_A < \eta_A^0$. We take sector A as an incumbent industry. After implementation of the regulation, the units $[0, \eta_A]$ stay in the market, while the most unproductive units $[\eta_A, \eta_A^0]$ exit the industry due to the rising cost burden.

We assume that the size of sector N is fixed and equals λ_0^0. Furthermore, it holds that

$$\Pi_0(\lambda_0^0) > 0.$$

The units in sector N choose between technologies zero and one. We assume that $\Delta\alpha \equiv \alpha_0 - \alpha_1 > 0$, so the emission level of one commodity unit production is lower using technology one than using technology zero. Hence, the cut-off unit λ_1 satisfies

$$\begin{gathered} \Pi_0(\lambda_1) = \Pi_1(\lambda_1) \iff \\ b_0 + \theta_0 - c_0\lambda_1 - s(\alpha_0 - l_0) = b_1 + \theta_1 - c_1\lambda_1 - s(\alpha_1 - l_1), \end{gathered} \tag{2.7}$$

and, in particular, if $s = 0$ (no regulation), then

[4]The terms supramarginal and inframarginal are used in the literature (Pezzey [54]). If $l_j < \alpha_j$, then the threshold is inframarginal, and if $l_j > \alpha_j$, then the threshold is supramarginal.

$$\Pi_0(\lambda_1^0) = \Pi_1(\lambda_1^0) \iff b_0 + \theta_0 - c_0\lambda_1^0 = b_1 + \theta_1 - c_1\lambda_1^0.$$

We will denote $\Delta l \equiv l_0 - l_1$, so we have

$$\lambda_1^0 = \frac{\Delta b - \Delta\theta}{\Delta c} \tag{2.8}$$

and

$$\lambda_1 = \frac{\Delta b - \Delta\theta + s\,(\Delta\alpha - \Delta l)}{\Delta c}. \tag{2.9}$$

If the regulation target is to drive the economy toward greener production, then we clearly should have $\lambda_1 > \lambda_1^0$. If this is the case, then the units $[0, \lambda_1]$ use green technology while the units $[\lambda_1, \lambda_0^0]$ use polluting brown technology. As compared to the choices without regulation, the units $\left[\lambda_1^0, \lambda_1\right]$ switch from brown to green technology. It may also be the case that $\lambda_1^0 = 0$. This means that green technology becomes economically viable only because of the environmental regulation.

Figure 2.1 illustrates the industry. The line $B_j(\lambda)$ corresponds to the technology j business-as-usual profits, while the line $\Pi_j(\lambda)$ involves the influence of the regulation, $j = 0, 1, A$. In drawing the sectors, we use solid lines to describe benefits and non-solid lines to describe final profits (including payments). In particular, note how our assumptions about Δb and Δc determine the structure of sector N. The assumptions mean in practice that firms apply green technology at the low end of the distribution. Figure 2.1(b) also shows how the utilization of green technology increases in sector N as a result of the regulation. In general, by Equations (2.8) and (2.9),

$$\lambda_1 - \lambda_1^0 = s\,(\Delta\alpha - \Delta l),$$

so the utilization of green technology increases in sector N as long as

$$\Delta\alpha - \Delta l > 0. \tag{2.10}$$

This condition is equivalent to

$$\alpha_0 - l_0 > \alpha_1 - l_1. \tag{2.11}$$

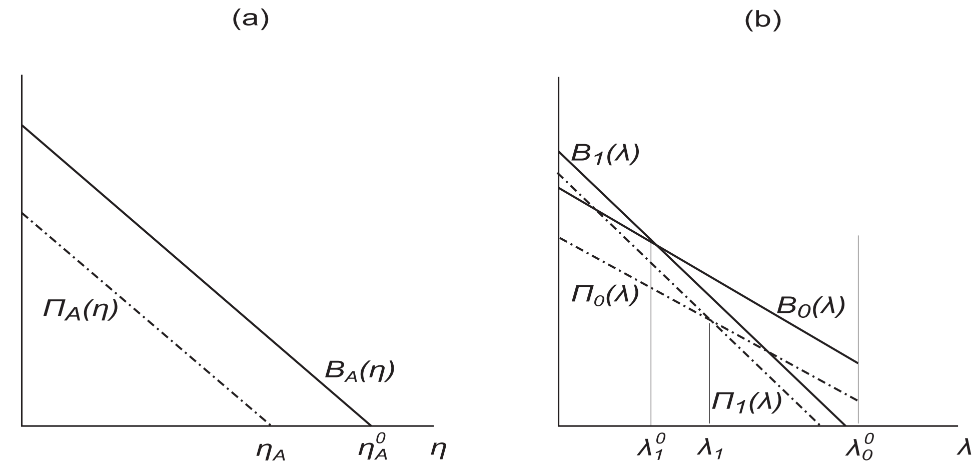

Figure 2.1 The Impact of Regulation. Various cut-off values are determined in sector A (a) and in sector N (b).

Remember that $\alpha_0 > \alpha_1$, so that technology zero is the dirty brown technology. Consequently, Condition (2.11) says that the agency should subsidize green technology more than brown technology. In Figure 2.1(b), the regulation shifts the profit lines inward, so we have $l_j < \alpha_j$, where $j = 0, 1$. On the other hand, if $l_0 < \alpha_0$ and $l_1 > \alpha_1$, the regulation shifts the profit line $\Pi_0(\lambda)$ inward and the profit line $\Pi_1(\lambda)$ outward, and the Condition (2.11) automatically holds. Overall, the condition $\Delta\alpha - \Delta l > 0$ is a sensible minimum target of the policy, as it induces strictly positive emission reductions in sector N.

Finally, by Figure 2.1(a), we see how $\eta_A^0 > \eta_A$, so the regulation induces some low-profit firms to exit from sector A. By Equation (2.6), this happens as long as $\alpha_A - l_A > 0$.

2.2.2 Perfect Participation

A public agency regulates the polluting industry because pollution creates harm and no voluntary bargain between the emitters and the sufferers has thus far been successful. The instrument used in regulation is either that of environmental taxation or a system of tradable permits. Furthermore, an important feature in the framework is the leader–follower set-up. Both the regulatory agency and the firms face the same uncertainty. The crux of the matter is that the agency sets the regulatory parameters before everyone learns of the uncertainty, and the agency is unable to re-optimize subsequently. Firms make all their choices only after the regulation is fixed and after everyone has learned of the uncertainty.

We first study the standard model of regulation, namely, the case in which participation is perfect. We assumed above that technology-specific pollution intensities are different between the technologies. If we denote the emissions in sector A and N by e_A and e_N, respectively, then

$$e_A = \int_0^{\eta_A} \alpha_A d\lambda = \alpha_A \eta_A \tag{2.12}$$

and

$$e_N = \int_0^{\lambda_1} \alpha_1 d\lambda + \int_{\lambda_1}^{\lambda_0^0} \alpha_0 d\lambda = \alpha_0 \lambda_0^0 - \Delta\alpha \lambda_1. \tag{2.13}$$

We take the pollution as homogenous in nature, so the total level of pollution

$$e = e_A + e_N \tag{2.14}$$

will be of interest. We assume that quadratic damage function reflects the harm, so the damage is

$$D(e) = \frac{d}{2} e^2, \tag{2.15}$$

where $d > 0$.[5] Furthermore, given the industry as specified above, we write the benefits as

$$B = \int_0^{\eta_A} B_A(\lambda) d\lambda + \int_0^{\lambda_1} B_1(\lambda) d\lambda + \int_{\lambda_1}^{\lambda_0^0} B_0(\lambda) d\lambda. \tag{2.16}$$

Alternatively, we can represent the benefits in terms of emissions. In this case, we denote the benefits by $B(e)$ and observe that this function is labeled as an abatement cost function in the literature. It can be shown that the abatement cost function is quadratic here and that

$$\gamma = \frac{\Delta c c_A}{c_A (\Delta\alpha)^2 + \Delta c \alpha_A^2} \tag{2.17}$$

[5] We assume the damage function is known with certainty. For more about this assumption, see Chapter 1, footnote 8.

is the slope of the marginal abatement function.[6] Finally, for notational convenience, we write

$$B_U = \int_0^{\eta_A^0} B_A(\lambda)d\lambda + \int_0^{\lambda_1^0} B_1(\lambda)d\lambda + \int_{\lambda_1^0}^{\lambda_0^0} B_0(\lambda)d\lambda \tag{2.18}$$

and

$$U = \int_0^{\eta_A^0} \alpha_A d\lambda \int_0^{\lambda_1^0} \alpha_1 d\lambda + \int_{\lambda_1^0}^{\lambda_0^0} \alpha_0 d\lambda. \tag{2.19}$$

We call these the counterfactual benefits and counterfactual emissions, respectively.

We introduce an “unrestricted" agency in this opening section. The entire polluting industry participates in the regulation, and the policy does not apply subsidies. Consequently, following the discussion in Chapter 1, the threshold policy $l_A = l_0 = l_1 = 0$ induces efficient emission allocation.[7] The regulator sets the strictness of the policy by choosing a proper tax rate. By Equations (B.1) and (B.2) in Appendix B.1, the benefits and emissions can be written as a function of the unit price s as

$$B(s) = B_U - \frac{1}{2\gamma}s^2$$

and

$$e(s) = U - \frac{1}{\gamma}s,$$

where B_U and U are defined in Equations (2.18) and (2.19), respectively. Denote the optimal rate by τ, so the necessary first-order condition requires that

$$E\left[\frac{dB}{d\tau} - \frac{dD}{de}\frac{de}{d\tau}\right] = 0,$$

or, that

$$\tau = dE[e]. \tag{2.20}$$

The derived policy rule is a standard one. It equates the price of the emissions to the

[6]We derived an abatement cost function earlier in Section 1.3.1.

[7]Chapter 1 also shows that an efficient allocation does not necessarily require zero subsidies. We will discuss this issue later in this chapter.

expected level of marginal damage.

If the agency implements the permit policy, it fixes first the number of pollution permits at level l and then auctions them off. At market equilibrium, the demand and supply of permits are equal, so

$$e_A + e_N = l.$$

The equilibrium permit price[8] that satisfies this condition is

$$p = \bar{p} + \gamma \left(\frac{\alpha_A}{c_A} \theta_A + \frac{\Delta\alpha}{\Delta c} \Delta\theta \right). \tag{2.21}$$

Especially note how a green industrial shock ($\Delta\theta < 0$) moves the permit price down. As far as the strictness of the permit policy is concerned, it is determined as in Chapter 1 (Section 1.3.2.2). The expected price $\bar{p}$ is taken as the choice variable, and it should be set to satisfy

$$\bar{p} = \tau.$$

The number of permits l is adapted to meet this price.

The other choice concerns the instrument in the regulation. Different instruments will induce different responses but there is a certain kind of symmetry between these responses. On one hand, the price is variable with tradable permits (Equation (2.21)), while in the tax system, the price is truly fixed. On the other hand, as

$$e(\tau) = l + \frac{\alpha_A}{c_A} \theta_A + \frac{\Delta\alpha}{\Delta c} \Delta\theta,$$

the quantity is variable with taxes, while in the permit system, the quantity is fixed and equal to l. Most importantly, even though it holds that $E(p) = \tau$ and $E(e(\tau)) = l$, the expected welfares between instruments may differ.

Let us define the comparative advantage between instrument I and J as

$$\Delta(I,J) = E\left[(B(I) - D(e(I))) - (B(J) - D(e(J))) \right]. \tag{2.22}$$

In our model,

[8] A note about the notation: We use a bar to indicate expected value.

$$EB(s) = EB_U - \frac{1}{2\gamma}E\left[s^2\right],$$

$$ED(e(\tau)) = \frac{d}{2}l^2 + \frac{d}{2}E\left(\frac{\alpha_A}{c_A}\theta_A + \frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2,$$

and

$$D(e(p)) = \frac{d}{2}l^2.$$

Based on these, it is easy to calculate that

$$\Delta(\tau, p) = \frac{1}{2}\frac{Var(p)}{\gamma^2}(\gamma - d). \tag{2.23}$$

The comparison follows the so-called Weitzman principle. It holds that the slopes of the marginal benefit γ and damage d functions solely determine the choice between the price (environmental tax) and the quantity (tradable permits). Thus, the choice between the instruments is immune to the size of the uncertainty. Note also that the comparison yields the standard Weitzman result, despite the assumed technology. The standard result in instrument choice holds, even though we assume technology transitions in sector N.[9] Note also that the assumption of zero subsidies is important. In Chapter 1, we study a policy of perfect participation that uses positive thresholds. We find that this feature of the policy may strongly influence the comparative advantage $\Delta(\tau, p)$. We will return to this issue toward the end of this chapter after we have studied imperfect participation first.

2.2.3 Imperfect Participation

We start our study of imperfect participation by studying a policy without voluntary provision. The policy regulates only sector A, and the units in sector N stay completely outside of the regulation. The benefits are

$$B_a = \int_0^{\eta_A} B_A(\lambda)d\lambda + \int_0^{\lambda_1^0} B_1(\lambda)d\lambda + \int_{\lambda_1^0}^{\lambda_0^0} B_0(\lambda)d\lambda$$

[9] In other words, the result holds, even if the marginal benefit function is only piece-wise linear in terms of the units (see Figure 2.1(b)).

while the emissions equal

$$e_a = \int_0^{\eta_A} \alpha_A d\lambda + \int_0^{\lambda_1^0} \alpha_1 d\lambda + \int_{\lambda_1^0}^{\lambda_0^0} \alpha_0 d\lambda.$$

We define units η_A, λ_1^0, and λ_0^0 in Section 2.2.1. As for the aggregate benefits of emissions, we write

$$B(e_A, e_N) = B_A(e_A) + B_N(e_N).$$

Specifically,

$$\frac{d^2 B_A(e_A)}{d e_A^2} = \gamma_a,$$

where

$$\gamma_a = \frac{c_A}{\alpha_A^2}. \tag{2.24}$$

The details of the regulation resemble those of the perfect participation. Consequently, the regulator either fixes the tax rate τ_a or the number of the auctioned permits l_a. According to Equations (B.7) and (B.8) in Appendix B.1, the benefits and the emissions as a function of the price level can be written as

$$B(s_a) = B_U - \frac{s_a^2}{2\gamma_a}$$

and

$$e(\tau_a) = U - \frac{1}{\gamma_a} s_a,$$

where B_U and U are defined in Equations (2.18) and (2.19), respectively. At the optimum, the tax rate (and the expected price level $\bar{p}_a$) satisfies

$$\tau_a = dE\left[e(\tau_a)\right], \tag{2.25}$$

so, in terms of the parameters of the model, we have

$$\tau_a = \frac{\gamma_a d}{\gamma_a + d} EU. \tag{2.26}$$

Furthermore, we have

$$\tau_a > \tau \text{ and } l_a > l$$

(see Appendix B.2, Part I), where τ and l are the policy choices with perfect participation. We note that we use factors

$$u \equiv \frac{c_A}{\Delta c}, a \equiv \frac{\Delta \alpha}{\alpha_A}, \tag{2.27}$$

and

$$n = 1 + ua^2 \tag{2.28}$$

in Appendix B.2. These factors are important as we will apply them repeatedly in the forthcoming sections.

Let us consider the policy choice more closely for a moment. First, note that (by assumption) the restricted regulator takes the scope of an environmental policy as given. Had she the opportunity to choose the scope, the regulator would prefer perfect participation (Appendix B.2, Part II). Second, the imperfection alters the nature of the abatement cost function. The function is only partial in nature, as some profitable abatement projects remain outside the set of plausible projects. Third, the policy calculations reflect the imperfect participation. The agency can only influence the emissions of sector A, so the emissions e_N merely enter the calculations as an exogenously given entity. Regarding the strictness of the policies, the shift in the marginal abatement cost function explains the observed differences.[10] However, the traditional first-best externality pricing rule $Es_a = dE[e(s_a)]$ holds even if participation is not perfect.

In the permit markets, the supply of permits equals the demand, or, equivalently,

$$e_A = l_a, \tag{2.29}$$

so the equilibrium price satisfies

$$p_a = \tau_a + \gamma_a \frac{\alpha_A}{c_A} \theta_A. \tag{2.30}$$

[10]In terms of emission reductions, the exclusion of sector N effectively means that the marginal abatement curve shifts upward at every level of emission reduction (see Figure 1 in Montero [47]). As marginal damages are increasing, the price and level of regulated emissions are increasing as well.

As for the aggregate emissions, we have

$$e(p_a) = l_a + e_N, \tag{2.31}$$

so

$$e(p_a) = l_a + E[e_N] - \frac{\Delta\alpha}{\Delta c}\Delta\theta.$$

Sector N remains outside the cap, so the emissions are uncertain with tradable permits. As for the tax policy, we have emissions equal to

$$e(\tau_a) = l_a + E[e_N] + \frac{\alpha_A}{c_A}\theta_A - \frac{\Delta\alpha}{\Delta c}\Delta\theta.$$

We define the comparative advantage Δ in Equation (2.22). We enter the various prices and quantities from above into this formula and write

$$\Delta(\tau_a, p_a) = \frac{1}{2}\frac{Var(p_a)}{\gamma_a^2}(\gamma_a - d). \tag{2.32}$$

In principle, less-than-full participation in the environmental program maintains the basic rule of instrument choice.[11] If the slope of the (effective) marginal benefit function exceeds the slope of the marginal damage function, then the price instrument should be chosen. We can also show that, as long as the permits are auctioned off, the selection of the regulated sector does not matter; that is, if sector N is the sole regulated industry, the rule in Equation (2.32) remains intact.

Fundamentally, we are interested in studying changes between different comparative advantages. In the present case, we seek differences between perfect and imperfect participation. In theory, we have two potential directions for change. The participation in regulation may turn from partial towards perfect or *vice versa.* The influence in instrument choice depends on the direction of the change: The quantity instrument will be favored as the regulatory regime shifts from imperfect to perfect participation. Conversely, the imperfection favors the price instrument.

The reason for these changes lies on the benefit side. As we are mainly interested in regulation that increases participation in regulation, we briefly discuss this case

[11]The analysis rules out any covariance between the random variables by assumption. Interestingly, the presence of covariance may induce differences between perfect and imperfect participation. In fact, regarding instrument choice, Weitzman [84] discusses the influence of covariance. See also Stavins [73] and Meunier [43].

here. If a voluntary provision succeeds in attracting new cost-saving projects into regulation, it will result in lower total and marginal cost curves. Following the basic principles of instrument choice, the less steep marginal cost curve means that more weight will be given to the slope of marginal damage d. In other words, as the relative importance of pollution damages increases, the relative importance of quantity control increases. More formally, it is straightforward to show that Equations (2.17), (2.24), and (2.28) together yield

$$\gamma_a = \gamma n,$$

where (by Equation (2.28)) $n > 1$, so $\gamma_a > \gamma$. So, we may rewrite the comparative measure under perfect participation (Equation (2.23)) as

$$\Delta(\tau, p) = \frac{Z}{2} \frac{Var(p_a)}{\gamma_a^2} (\gamma_a - nd), \tag{2.33}$$

where

$$Z = \frac{1 + (ua)^2}{1 + ua^2} > 0. \tag{2.34}$$

The multiplier $n > 1$ is the additional weight given to the slope of the marginal damage.[12] For example, let $\gamma_a = d$, so (by Equation (2.32)) it holds that $\Delta(\tau_a, p_a) = 0$. The regulator is indifferent between prices and quantities under imperfect participation. If participation becomes perfect (and the policy choices become optimal alongside it), then (by Equation (2.33)) $\Delta(\tau, p) < 0$ and the quantities become the preferred choice. We call the multiplier n the scope effect in instrument choice.[13]

[12] The size of factor Z is irrelevant for instrument choice, as it only magnifies the size of the measure Δ.

[13] It also holds that

$$\Delta(\tau_a, p_a) = \frac{1}{2} \frac{1}{Z} \frac{Var(p)}{\gamma^2} (\gamma n - d).$$

That is, the price instrument will be favored as the regulatory regime shifts from perfect to imperfect participation.

2.3 Regulation with Voluntary Participation

2.3.1 The Industry

Above, we studied policy that regulates only a fraction of the polluting firms. The agency took this state as exogenously given. In the following sections, the agency will change the policy target: As the mandatory approach does not turn out to be successful in sector N, then the agency will try the voluntary approach instead.[14] In terms of environmental payments, the environmental fees do not work in sector N. Thus, in the implementation of the voluntary provision, the agency must apply subsidies in sector N instead.

The non-affected sector N is denoted by NA as some units in sector N become involved in regulation. The polluting industry then consists of two distinct sectors, the affected sector A and the non-affected sector NA, and the agency takes this distinction as a given. As mentioned earlier, firms in the affected sector face mandatory environmental regulation, while firms in the non-affected sector do not. The participation is then imperfect. However, voluntary participation is now plausible, so firms in sector NA may voluntarily become affected.

Participation is not a random process but is based on economic incentives. A unit in sector NA decides whether to participate and, if it participates, whether to switch to green technology. The functions $B_i(\lambda)$ and $\Pi_i(\lambda)$ (in Equations (2.2) and (2.3)) above describe the various consequences. More specifically, with respect to participation, it must hold that $\Pi_i(\lambda) \geq B_j(\lambda)$ at least for one technology i, where $j = 0, 1$. These conditions simply state that the participation should raise the current profits. With respect to the switch in technology, the condition $\Pi_1(\lambda) \geq \Pi_0(\lambda)$ should apply.

To be profitable, voluntary participation invariably requires that $P_i \leq 0$, $i = 0, 1$. To satisfy this, the policy must have $l_i \geq \alpha_i$. Voluntary participation requires that the policy employs supramarginal thresholds. However, it does not necessarily follow that every firm will participate, nor that every firm will use the same technology in sector NA. Within this diversity of possibilities, Table 2.1 collects the various participation regimes and the corresponding conditions for determining the cut-off units η_A and λ_1. Especially note how the three regimes depend on the sizes of thresh-

[14]This view naturally presupposes that society benefits from increasing participation. We will return soon to this issue below.

Table 2.1 The Responses

$l_1 \geq \alpha_1, l_0 \geq \alpha_0$ Brown and green participate
$\begin{cases} \Pi_A(\eta_A) = 0 \\ \Pi_1(\lambda_1) = \Pi_0(\lambda_1) \end{cases}$ $\begin{cases} B_A(\eta_A) = s(\alpha_A - l_A) \\ B_0(\lambda_1) - B_1(\lambda_1) = s(\Delta\alpha - \Delta l) \end{cases}$
$l_1 \geq \alpha_1$ and $l_0 < \alpha_0$ Only green participates
$\begin{cases} \Pi_A(\eta_A) = 0 \\ \Pi_1(\lambda_1) = B_0(\lambda_1) \end{cases}$ $\begin{cases} B_A(\eta_A) = s(\alpha_A - l_A) \\ B_0(\lambda_1) - B_1(\lambda_1) = s(l_1 - \alpha_1) \end{cases}$
$l_1 < \alpha_1$ and $l_0 \geq \alpha_0$ Only brown participates
$\begin{cases} \Pi_A(\eta_A) = 0 \\ \Pi_0(\lambda_1) = B_1(\lambda_1) \end{cases}$ $\begin{cases} B_A(\eta_A) = s(\alpha_A - l_A) \\ B_0(\lambda_1) - B_1(\lambda_1) = s(l_0 - \alpha_0) \end{cases}$

olds, so they will depend on the details of subsidization. Note also that in our analysis below, the environmental agency does not determine the subsidization but rather takes it as exogenously given. Environmental agency is a restricted agency, so it does not choose the regime but rather takes the regime as given.

For example, consider the policy in the middle of Table 2.1, that is, the policy where only green technology will participate. Based on the low end of the middle section, the cut-off units are

$$\eta_A = \frac{b_A + \theta_A - s(\alpha_A - l_A)}{c_A} \text{ and } \lambda_1 = \frac{\Delta b - \Delta\theta + s(l_1 - \alpha_1)}{\Delta c}. \tag{2.35}$$

Figure 2.2 illustrates the polluting industry. In Figure 2.2(b), the units $[0, \lambda_1]$ participate in the voluntary program and use green technology, while the units $[\lambda_1, \lambda_0^0]$ do not participate and continue to use polluting brown technology. It holds that $0 < \lambda_1 < \lambda_0^0$, so technologies zero and one exist side by side at the industry equilibrium. Note also that the units $\left[\lambda_1^0, \lambda_1\right]$ switch from brown to green technology as a

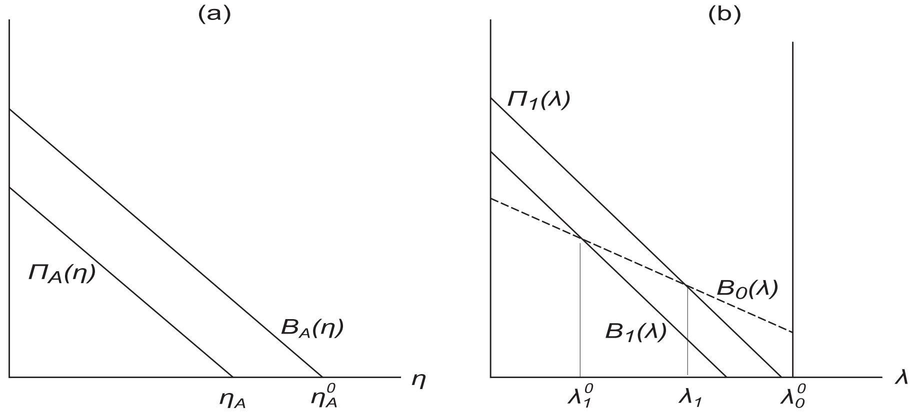

Figure 2.2 Voluntary Participation. A case in which only green technology participates. Various cut-off values are determined in sector A (a) and in sector NA (b).

result of the voluntary provision.

2.3.2 Efficient Allocation

A conspicuous new feature in the analysis of imperfect participation is the use of subsidies. The use of supramarginal thresholds becomes a necessity when the agency pursues greater participation. Let us discuss the fundamental changes in social welfare that voluntary participation creates. We start with the aggregate benefits across technologies. For this purpose, we set the stochastic variables equal to zero for a moment and write the optimization problem under certainty as

$$\max_{\eta_A, \lambda_1} \int_0^{\eta_A} B_A(\lambda) d\lambda + \int_0^{\lambda_1} B_1(\lambda) d\lambda + \int_{\lambda_1}^{\lambda_0^0} B_0(\lambda) d\lambda$$
$$s.t.\ e = \alpha_A \eta_A + \alpha_0 \lambda_0^0 - \Delta\alpha\lambda_1.$$

If we denote the Lagrange multiplier by μ, the solution requires that

$$\begin{cases} B_A(\eta_A) - \mu\alpha_A = 0 \\ B_1(\lambda_1) - B_0(\lambda_1) + \mu\Delta\alpha = 0 \end{cases}. \tag{2.36}$$

Consequently, the satisfaction of these rules guarantees an efficient emission allocation. Furthermore, if we enter the efficient units back to the objective function, we have an abatement cost function $B(e)$.

Efficient implementation means that we specify the values l_A, l_0 and l_1 in such a way that they will lead the economy towards an efficient allocation of emissions. In this exercise, we need technology-specific response functions across industries. These functions are calculated in the lower parts of the cells in Table 2.1. Consequently, by incorporating *efficient thresholds* l_A, l_0 and l_1 into these reaction functions, an efficient allocation of emissions (implied by Equations (2.36)) follows.

First, assume that both technologies will participate. Clearly, the efficiency will follow if

$$\frac{l_A}{\alpha_A} = \frac{\Delta l}{\Delta \alpha}. \tag{2.37}$$

For example, assume that the units in the affected sector have to purchase every single permit from the auction. Then $l_A = 0$ and the efficiency will follow as long as $l_0 = l_1$.[15] More generally, by letting $l_A > 0$, the efficient allocation requires that $\Delta l > 0$. It requires that the agency subsidize brown technology users more heavily than green polluters. Furthermore, the more firms in sectors A are subsidized, the more brown technology users should be subsidized (in relative terms). These efficiency arguments may well raise some distributional and political debates. In particular, if the size of the threshold l_A is non-negotiable, the efficient subsidization might be considered as too extreme a solution and will be rejected. Nevertheless, the rule in Equation (2.37) is an example of the relative incentives that we already discussed in Chapter 1 within a two-sector model.[16]

In the second case of interest, only green technology participates (middle part in Table 2.1).[17] An efficient implementation means that an equality

$$\frac{\alpha_A - l_A}{\alpha_A} = \frac{l_1 - \alpha_1}{\Delta \alpha} \tag{2.38}$$

should hold. In particular, if $l_A = 0$, then efficiency requires that $l_1 = a_0$. More

[15]Furthermore, it must hold that $l_0 (= l_1) \geq \alpha_0$.

[16]It can be shown that the rule in Equation (2.37) also applies in the case of perfect participation. In that context, the principle that polluters pay ($l_A = l_0 = l_1 = 0$) holds as a specific efficient solution.

[17]It holds that $l_0 < \alpha_0$ under this particular subsidization regime. Consequently, the brown units do not participate. The participation then solves $\Pi_1(\lambda) = B_1(\lambda)$.

generally, the efficient implementation implies (utilize Equations (2.35)) that

$$\lambda_1 = \frac{\Delta b + s\Delta\alpha\left(1 - \frac{l_A}{\alpha_A}\right)}{\Delta c}.$$

Even though we subsidize brown production in the affected sector ($l_A > 0$), we have $\lambda_1 > \lambda_1^0$, so the efficiency always induces an increase in the use of green technology.[18] Note also that, if $l_A > 0$, then the efficient allocation means that $l_1 < a_0$. Whenever firm-specific subsidization in the affected sector increases, the efficient solution requires that firm-specific subsidization of the voluntary participants decrease. Furthermore, note that an efficient solution does not automatically require perfect participation. It is quite possible that $\lambda_1 < \lambda_0^0$, so that not every firm voluntarily participates in an efficient solution.

In the third and final case, the policy induces only brown technology units to participate (bottom part in Table 2.1). We see that efficient allocation will not happen. In fact, efficiency calls for the brown firms in sector N to turn towards greener production, so the mere subsidization of brown production cannot implement this goal.

2.3.3 Restricted Choice

We do not take efficient implementation as granted. Rather, we will incorporate into analysis the same assumption as in Chapter 1. The environmental agency does not determine the thresholds l_A, l_0, and l_1. We assume that the thresholds do not necessarily reflect the efficiency view.

We will consider the first and second cases discussed above. These policies are able (at least in principle) to implement efficiency. We do not pursue the most general representation but rather somewhat restrict the set of plausible thresholds. In both cases, we will set $l_A = 0$ from the outset, so the firms in sector A buy every single license they use, or pay the tax for every emission unit that they produce. Relaxing the assumption $l_A = 0$ is discussed at the end of the chapter.

The first case questions the outcome $l_0 > \alpha_0$, so it must hold that $l_0 \leq \alpha_0$ instead. The actual policy cannot subsidize brown production. This implies that a voluntary participant never uses brown technology. In fact, we do not sustain a positive l_0 at

[18] We are interested in solutions in which the environmental regulation causes the level of aggregate emissions to decrease. Then, it must be the case that $l_A < \alpha_A$.

all, so we take $l_0 = 0$ instead. Regarding green production, we do not fix a certain value but assume a continuum of plausible values. However, we set $l_1 - \alpha_1 \geq 0$, so some firms do participate and reduce emissions upon participation.[19]

In the second case, we reject the assumption $l_0 \leq \alpha_0$ and choose $l_0 > \alpha_0$ instead. This means that the entire brown industry will participate. This may sound like a useless and even harmful policy especially in situations where the subsidization of firms is difficult. After all, subsidization of brown technology as such induces no pollution reductions that are the main purpose of the voluntary provisions. One beneficial consequence of the policy is that it may enclose the entire sector NA inside the regulation. We show how this feature will have a substantial effect on the regulatory outcome. However, we require that the voluntary participation provision has to induce some firms to switch from brown to green technology. Referring to our earlier discussion (see especially Condition (2.11)), this policy requires that $\Delta\alpha - \Delta l \geq 0$.

The cut-off units then become

$$\eta_{Am} = \frac{b_A + \theta_A - s_m \alpha_A}{c_A} \text{ and } \lambda_{1m} = \frac{\Delta b - \Delta\theta + s_m \varphi_m}{\Delta c}, \tag{2.39}$$

where $m = 1, 2$. The subscript m refers to different implementations: either only green technology participates ($m = 1$) or both green and brown technologies participate ($m = 2$). Specifically,

$$\varphi_1 = l_1 - \alpha_1 \geq 0 \tag{2.40}$$

and

$$\varphi_2 \equiv \Delta\alpha - \Delta l \geq 0. \tag{2.41}$$

The factor φ_1 applies whenever only green participates, while φ_2 applies if both colors participate. The non-negativity of φ_1 and φ_2 follows from our assumption above. The positive values imply that voluntary provision induces positive emission reductions.

[19]We have assumed that every firm that utilizes technology one will be subsidized. This assumption simplifies the presentation without affecting the quality of our main results.

2.3.4 Instrument Design

2.3.4.1 Second-Best Policy

The restricted regulator now takes the voluntary participation era and the (possibly inefficient) threshold distribution (l_A, l_0, l_1) as given and chooses the strictness of the policy and the instrument to be applied. In this approach, we incorporate the cut-offs (Equations (2.39)) into the benefits (Equation (2.16)) and into the damages (Equations (2.12)—(2.15)), and differentiate the expected social welfare with respect to the tax rate τ_m. We do this in Appendix B.1 (part Voluntary Participation, Equation (B.13)). It holds that

$$\tau_m = \frac{d\gamma_m^l \gamma_m^L}{\left(\gamma_m^L\right)^2 + d\gamma_m^l} EU, \tag{2.42}$$

where

$$\gamma_m^l = \frac{c_A \Delta c}{\Delta c \alpha_A^2 + c_A (\varphi_m)^2}, \tag{2.43}$$

$$\gamma_m^L = \frac{c_A \Delta c}{\Delta c (\alpha_A)^2 + c_A \Delta \alpha \varphi_m}, \tag{2.44}$$

EU is the expected counterfactual, and $m = 1, 2$. The factors γ_m^l and γ_m^L turn out to be slope coefficients in certain marginal benefit functions.

We present three points about efficient solutions. These are:

i. In an efficient solution, $\gamma_m^l = \gamma_m^L$, $m = 1, 2$.

ii. If the solution is efficient, we have $\tau_1 = \tau_2$.

iii. The rule $\tau_m = dE[e(\tau_m)]$ (corresponding to earlier rules in Equations (2.20) and (2.25)) holds only if the solution is efficient.

We can prove the first point by noting (by Equations (2.37) and (2.38)) that the condition $\varphi_m = \Delta\alpha$ must hold at efficient solutions. The second point follows, as we incorporate the condition $\gamma_m^l = \gamma_m^L$ into the definition of τ_m (Equation (2.42)). Thirdly, we use the value of $e(\tau_m)$ (Equation (B.11) in Appendix B.1) when we write

$$dE(e(\tau_m)) - \tau_m = dEU - \frac{d + \gamma_m^L}{\gamma_m^L} \tau_m.$$

After incorporating τ_m from Equation (2.42),

$$dE(e_m)-\tau_m = dEU\left(1-\frac{\frac{\gamma_m^l}{\gamma_m^L}(\gamma_m^L)^2+d\gamma_m^l}{(\gamma_m^L)^2+d\gamma_m^l}\right) = dEU\frac{(\gamma_m^L)^2}{(\gamma_m^L)^2+d\gamma_m^l}\left(1-\frac{\gamma_m^l}{\gamma_m^L}\right).$$

Clearly, $\tau_m \neq dE[e(\tau_m)]$ as long as $\gamma_m^l \neq \gamma_m^L$, where $m = 1,2$. As a corollary, we can write the general relationship between the tax rate and expected emissions as

$$\tau_m = \frac{\gamma_m^l}{\gamma_m^L} dE[e_m], \tag{2.45}$$

where $m = 1,2$.

2.3.4.2 Differences Between Optimal Expected Prices and Quantities

We are interested in implementation of voluntary participation that increases social welfare. We show in next section that this objective can be expressed as a condition

$$n > \rho_m, \tag{2.46}$$

where n is the scope effect defined above in Equation (2.28) and $m = 1,2$. Factor ρ_m, called a cost effect, contains the influence of the inefficient subsidization. It is shown below that $\rho_m > 1$ as long as the subsidization is inefficient and that $\rho_m = 1$ under efficiency. Remember that the scope effect describes the benefits that the inclusion of new cost-saving projects create in the regulated industry. In this respect, cost effect displays the costs of these projects and records the fact that inefficient subsidies are employed to attract new projects. The Condition (2.46) then presents an intuitive idea that social welfare increases if benefits exceed the costs.

We state next the various results concerning optimal expected prices and quantities (see Appendix B.2, Part III):

Lemma 1 *Let us denote*

$$k_m \equiv \frac{\varphi_m}{\alpha_A} \geq 0, \tag{2.47}$$

where $m = 1,2$. *Then,*

i. The sign of the difference $\tau_m - \tau$ *depends on the sign of* $a - k_m$. *Specifically, if*

$k_m > 0$ and $a - k_m = 0$, then $\tau_m - \tau = 0$.

ii. *The sign of the difference $\tau_m - \tau_a$ depends on the sign of $a - k_m$. Specifically, if $k_m = 0$, then $\tau_m - \tau_a = 0$.*

iii. *$E[e(\tau_m)] - E[e(\tau)] > 0$ as long as $\rho_m > 1$. If $\rho_m = 1$, then $E[e(\tau_m)] - E[e(\tau)] = 0$.*

iv. *$E[e(\tau_a)] - E[e(\tau_m)] > 0$ as long as $n > \rho_m$. If $n = \rho_m$, then $E[e(\tau_a)] - E[e(\tau_m)] = 0$.*

We denote emissions under perfect participation by $e(\tau)$, while $e(\tau_a)$ and $e(\tau_m)$ represents emissions under zero and strictly positive voluntary participation, respectively. In general, the sign differences between different optimal tax rates cannot be nailed down unambiguously. We derived similar results in Chapter 1. Note further that, by Equations (2.37) and (2.38), efficiency implies that $a - k_m = 0$. Thus, under efficiency, the optimal tax rate τ_m equals the perfect participation rate τ. On the other hand, zero voluntary participation means that $k_m = 0$, so $\tau_m = \tau_a$ under zero participation.[20]

The differences between optimal expected emissions are essential in the analysis to come. First, the emissions under voluntary participation strictly exceed the emissions under perfect participation ($E[e(\tau_m)] > E[e(\tau)]$) as long as the subsidization is inefficient ($\rho_m > 1$) and they are equal under efficiency ($\rho_m = 1$). Second, the voluntary participation decreases emissions ($E[e(\tau_a)] > E[e(\tau_m)]$) as long as $n > \rho_m$. If no firm participates, then $n = \rho_m$ and $E[e(\tau_a)] - E[e(\tau_m)]$.

We claim (see Condition (2.46)) above that the scope effect should exceed the cost effect in a meaningful policy. In Appendix B.2, Part III, we write these effects in terms of parameters of the model. In particular, by Equation (B.21), we may write

$$\frac{n}{\rho_m} = \frac{(1 + uak_m)^2}{1 + uk_m^2},$$

where

$$u \equiv \frac{c_A}{\Delta c} > 0\,,\; a \equiv \frac{\Delta\alpha}{\alpha_A} > 0,\; k_m \equiv \frac{\varphi_m}{\alpha_A} \geq 0, \tag{2.48}$$

(see Equations (2.27) and (2.47)) and $m = 1, 2$. Consequently, we have $\frac{n}{\rho_m} > 1$ as long

[20] Note that the condition $k_2 = 0$ holds if voluntary participation equals zero ($l_0 - \alpha_0 = l_1 - \alpha_1 = 0$), but it may hold under positive participation as well ($l_0 - \alpha_0 = l_1 - \alpha_1 > 0$).

as the condition

$$1 + uak_m + \left(1 - \frac{k_m}{a}\right) > 0 \qquad (2.49)$$

holds. In particular, the condition $a > k_m$ is sufficient for a meaningful policy.[21]

2.3.4.3 Comparison of the Regulatory Regimes

The next question we pose is about the overall meaningfulness of the voluntary provision. After all, the imperfect implementation of voluntary participation induces inefficiencies, so it may also crucially reduce the benefits of the policy. As for the existence of this trade-off, we have:

Proposition 3 *If the environmental agency has the opportunity to choose the voluntary provision in place of perfect participation, it would still prefer perfect participation. On the other hand, assume that Condition (2.46) applies. Then the agency prefers strictly positive voluntary participation to zero voluntary participation.*

Proof. We calculate first the expected welfare under positive voluntary participation in Appendix B.1, Part Voluntary Participation. By Equation (B.14), it equals

$$EW_{\tau_m} = EB_U - \frac{\gamma_m^L}{\gamma_m^l}\tau_m\frac{EU}{2} - \frac{d}{2}E\Phi^2,$$

where

$$\Phi = a_A\frac{\theta_A}{c_A} - \Delta a\frac{\Delta\theta}{\Delta c}$$

and $m = 1,2$. The expected welfares under perfect participation and zero voluntary participations are (by Equations (B.6) and (B.9) in Appendix B.1)

$$EW_\tau = EB_U - \tau\frac{EU}{2} - \frac{d}{2}E\Phi^2$$

and

$$EW_{\tau_a} = EB_U - \tau_a\frac{EU}{2} - \frac{d}{2}E\Phi^2,$$

respectively. Note that the three expected welfares are calculated using the optimal tax rates. Note further that these rates are (by Equations (2.20), (2.25), and (2.45))

[21] Note that we have either $a - k_1 = \frac{\alpha_0 - l_1}{\alpha_A}$ or $a - k_2 \equiv \frac{\Delta l}{\alpha_A}$.

$$\tau = dE[e(\tau)],\ \tau_a = dE[e(\tau_a)],\ \text{and}\ \tau_m = \frac{\gamma_m^l}{\gamma_m^L} dE[e(\tau_m)],$$

where $m = 1,2$. Now,

$$\begin{aligned} EW_{\tau_m} - EW_{\tau} &= -\frac{\gamma_m^L}{\gamma_m^l}\tau_m \frac{EU}{2} + \tau \frac{EU}{2} = \left(\tau - \frac{\gamma_m^L}{\gamma_m^l}\tau_m\right)\frac{EU}{2} \\ &= d\left(E[e(\tau)] - E[e(\tau_m)]\right)\frac{EU}{2} \end{aligned}$$

and

$$EW_{\tau_m} - EW_{\tau_a} = -\frac{\gamma_m^L}{\gamma_m^l}\tau_m \frac{EU}{2} + \tau_a \frac{EU}{2} = d\left(E[e(\tau_a)] - E[e(\tau_m)]\right)\frac{EU}{2},$$

where $m = 1,2$. By Lemma 1, we have $E[e(\tau)] - E[e(\tau_m)] < 0$ and $E[e(\tau_a)] - E[e(\tau_m)] > 0$. As $d > 0$ and $EU > 0$, we have

$$EW_{\tau} > EW_{\tau_m} > EW_{\tau_a}, \tag{2.50}$$

where $m = 1,2$. ■

2.3.5 Prices and Quantities

2.3.5.1 Permit Implementations

Let us return to the implementation issues. The restricted agency implements the regulation either by environmental taxes or by tradable permits. In the implementation of the permit policy, it should deliver an appropriate number of permits. In general, if we denote the total number of permits in the market by L_m, then

$$L_m = l_m + \iota_1 \int_0^{\lambda_{1m}} l_1 d\lambda + \iota_2 \int_{\lambda_{1m}}^{\lambda_0^0} l_0 d\lambda = l_m + \iota_1 \lambda_{1m} l_1 + \iota_2 \left(\lambda_0^0 - \lambda_{1m}\right) l_0, \tag{2.51}$$

where l_m is the number of auctioned permits, $m = 1,2$ and symbols ι_1 and ι_2 are indicator variables.[22] We define them as

$$\iota_1 = \begin{cases} 0 \text{ if } l_1 \leq \alpha_1 \\ 1 \text{ if } l_1 > \alpha_1 \end{cases} \tag{2.52}$$

and

$$\iota_2 = \begin{cases} 0 \text{ if } l_0 \leq \alpha_0 \\ 1 \text{ if } l_0 > \alpha_0 \end{cases} . \tag{2.53}$$

Thus, ι_1 takes the value of one whenever units in green technology voluntarily participate, while ι_2 is one whenever units in brown technology voluntarily participate. Above, we ruled out the case where only units in brown technology participates. Then, ι_2 can take the value of one only if ι_1 takes the value of one. Conversely, ι_1 can be one even though ι_2 is zero because we allow only green technology to participate. In any case, the last part in Equation (2.51) stands for the "private permit supply" (i.e., the accumulation of the free initial allocation).

We write the permit market equilibrium as

$$\begin{aligned} l_m = \int_0^{\eta_{Am}} \alpha_A d\lambda - \iota_1 \int_0^{\lambda_{1m}} (l_1 - \alpha_1) d\lambda - \iota_2 \int_{\lambda_{1m}}^{\lambda_0^0} (l_0 - \alpha_0) d\lambda = \eta_{Am}(l_m)\alpha_A \\ - \lambda_{1m}(l_m)[\iota_2(m)(l_0 - \alpha_0) - \iota_1(m)(l_1 - \alpha_1)] - \iota_2(m)\lambda_0^0(l_0 - \alpha_0). \end{aligned} \tag{2.54}$$

The right-hand side gives the total demand, while the left-hand side is the supply of the auctioned permits. We insert first the values of η_{Am} and λ_{1m} from Equations (2.39) into the aggregate demand. After this, we solve the equilibrium price, which gives us

$$\begin{aligned} p_m = \gamma\left(\frac{b_A + \theta_A}{c_A}\alpha_A - \frac{\Delta b - \Delta\theta}{\Delta c}[\iota_2(m)(l_0 - \alpha_0) - \iota_1(m)(l_1 - \alpha_1)]\right) \\ - \gamma\left(\iota_2(m)\lambda_0^0(l_0 - \alpha_0) + l_m\right), \end{aligned} \tag{2.55}$$

[22] The units with different technologies enter in a lumpy fashion. That is, as l_0 becomes strictly greater than α_0, the number of $\lambda_0^0 - \lambda_1$ brown firms enter the voluntary program. Similarly, as l_1 becomes strictly greater than α_1, the number of λ_1 green firms enter the program.

where

$$\gamma = \left(\frac{c_A \Delta c}{\Delta c \alpha_A \alpha_A + c_A \varphi_m [\iota_2 (l_0 - \alpha_0) - \iota_1 (l_1 - \alpha_1)]} \right)$$

and $m = 1, 2$. Definitions (2.40), (2.41), (2.52), and (2.53) together allow us to write the permit price as

$$p_m = \frac{c_A \Delta c}{\Delta c \alpha_A^2 + c_A (\varphi_m)^2} \left(\frac{b_A + \theta_A}{c_A} \alpha_A - \frac{\Delta b - \Delta \theta}{\Delta c} \varphi_m \right) - \frac{c_A \Delta c}{\Delta c \alpha_A^2 + c_A (\varphi_m)^2} \left(\iota_2(m) \lambda_0^0 (l_0 - \alpha_0) + l_m \right),$$

or by Equation (2.43) as

$$p_m = \gamma_m^l \left(\frac{b_A + \theta_A}{c_A} \alpha_A - \frac{\Delta b - \Delta \theta}{\Delta c} \varphi_m - \iota_2(m) \lambda_0^0 (l_0 - \alpha_0) - l_m \right),$$

where $\iota_2(1) = 0$, $\iota_2(2) = 1$, and $m = 1, 2$. Finally, we denote the deterministic part of the price as $\overline{p}_m$, so

$$p_m = \overline{p}_m + \gamma_m^l \left(\frac{\theta_A}{c_A} \alpha_A + \frac{\Delta \theta}{\Delta c} \varphi_m \right), \tag{2.56}$$

where $m = 1, 2$.

The tax rate τ_m calculated in Equation (2.42) gives us the stringency of the tax policy. The stringency of the permit policy follows as we set $\overline{p}_m = \tau_m$ (see Chapter 1, Section 1.3.2.2). As for the other variable of interest, namely, the regulated level of emissions, we write first Equation (2.14) as

$$e(s_m) = \alpha_A \eta_{Am} + \alpha_0 \lambda_0^0 - \Delta \alpha \lambda_{1m}.$$

Next, we insert the values of η_{Am} and λ_{1m} from Equations (2.39) and find that

$$\begin{aligned} e(s_m) &= \alpha_0 \lambda_0^0 + \alpha_A \left(\frac{b_A + \theta_A - s_m \alpha_A}{c_A} \right) - \Delta \alpha \left(\frac{\Delta b - \Delta \theta + s_m \varphi_m}{\Delta c} \right) \\ &= EU + \alpha_A \left(\frac{\theta_A - s_m \alpha_A}{c_A} \right) - \Delta \alpha \left(\frac{-\Delta \theta + s_m \varphi_m}{\Delta c} \right), \end{aligned}$$

where EU is the level of expected counterfactual emissions. After arranging, it holds that

$$e(s_m) = EU + \alpha_A \frac{\theta_A}{c_A} + \Delta\alpha \frac{\Delta\theta}{\Delta c} - s_m \left(\frac{\Delta c\,(\alpha_A)^2 + c_A \Delta\alpha \varphi_m}{c_A \Delta c} \right),$$

or, by using Equation (2.44), it holds that

$$e(s_m) = EU + \alpha_A \frac{\theta_A}{c_A} + \Delta\alpha \frac{\Delta\theta}{\Delta c} - \frac{1}{\gamma_m^L} s_m. \tag{2.57}$$

Importantly, as far as $\overline{p}_m = \tau_m$, then

$$E\left[e(\tau_m)\right] = E\left[e(p_m)\right],$$

where $m = 1, 2$. Therefore, the two instruments will yield the same expected level of emissions.

2.3.5.2 Notational Issues

To facilitate the upcoming analysis, we develop some new notation. We will introduce parameters $R_0(s_m)$ and $R_1(s_m)$, where the price entry inside the parenthesis indicates that the parameters are specific to the implementation. We can now represent the unit price of emissions as

$$s_m = \overline{s}_m + \gamma_m^L \left(R_0(s_m) \frac{\alpha_A}{c_A} \theta_A + R_1(s_m) \frac{\Delta\alpha}{\Delta c} \Delta\theta \right). \tag{2.58}$$

With permit implementation,

$$R_0(p_m) = \frac{\gamma_m^l}{\gamma_m^L} \text{ and } R_1(p_m) = \frac{\varphi_m}{\Delta\alpha} R_0(p_m), \tag{2.59}$$

(see Equation (2.56)), while with tax implementation

$$R_0(\tau_m) = R_1(\tau_m) = 0, \tag{2.60}$$

where $m = 1, 2$. As for the emissions, define first

$$\overline{e}_m = EU - \frac{1}{\gamma_m^L} \overline{p}_m.$$

Then, we can rewrite the emissions (Equation (2.57)) as

$$e(s_m) = \bar{e}_m + (1 - R_0(s_m))\frac{\alpha_A}{c_A}\theta_A + (1 - R_1(s_m))\frac{\Delta\alpha}{\Delta c}\Delta\theta, \tag{2.61}$$

where $m = 1, 2$. The reader can verify that inserting the specific values from Equations (2.59) and (2.60) into this formula will indeed yield the correct emissions.

Depending on the implementation, it holds that $l_1 \in [\alpha_1, \alpha_0]$ ($m = 1$) or $\Delta l \in [0, \Delta\alpha]$ ($m = 2$). Furthermore, by using the definitions of a and u (Equations (2.27)), we may also write

$$R_0(p_m) = \frac{1 + uak_m}{1 + uk_m^2} \text{ and } R_1(p_m) = \frac{k_m}{a}R_0(p_m). \tag{2.62}$$

Since

$$R_0(p_m) = \frac{a}{k_m}\frac{1 + uak_m}{\frac{a}{k_m} + uak_m}$$

the magnitudes of R_0 and R_1 depend on the sizes of a and k_m.

Finally, let us consider the variables at the extremes. First, let $\frac{a}{k_m} = 1$. We know that this corresponds to an efficient allocation. It either holds that $l_1 = \alpha_0$ ($m = 1$) or $l_1 = l_0$ ($m = 2$). In either case, we have $R_0(p_m) = R_1(p_m) = 1$ and $\varphi_1 = \varphi_2 = \Delta\alpha$, so

$$p_m = \bar{p}_m + \gamma\left(\frac{\alpha_A}{c_A}\theta_A + \frac{\Delta\alpha}{\Delta c}\Delta\theta\right) \text{ and } e(p_m) = \bar{e}_m.$$

By our discussion above (see also Part i in Lemma 1),

$$p_m = \bar{p} + \gamma\left(\frac{\alpha_A}{c_A}\theta_A + \frac{\Delta\alpha}{\Delta c}\Delta\theta\right) \text{ and } e(p_m) = \bar{e}, \tag{2.63}$$

where $m = 1, 2$. In particular, the fluctuating emissions will disappear.[23]

Finally, if $l_1 = \alpha_1$ and $l_0 = \alpha_0$, the allocation is extremely tight, so not a single firm will opt in. This time, we have $k_m = 0$, so, by Equations (2.62), we have $R_0(p_m^l) = 1$

[23]We note two things about the efficient implementation here. First, the price under imperfect participation (p_m) behaves like a price under perfect participation (p). Thus, the participation of the brown technology is immaterial as long as the policy implements the same amount of green technology as the perfect participation induces. Second, the efficient prices p_1 and p_2 are the same as p even though the thresholds between the implementations differ. However, we discuss in Chapter 1 that different efficient thresholds will yield different efficient prices. The answer to this dilemma merely lies in the assumption $l_A = 0$. If we assume that $l_A > 0$ instead, then the efficient prices will differ similarly as they do in Chapter 1.

and $R_1(p_m^l)=0$, $m=1,2$. As participation is zero, then it also holds that $\iota_1=\iota_2=0$ (by Equations (2.52) and (2.53)). Writing Equations (2.55) and (2.57) out in full, we have $p_1=p_2=p_a$ and $e(p_1)=e(p_2)=e_a$. Consequently,

$$p_m=p_a=\overline{p}_a+\gamma_a\frac{\alpha_A}{c_A}\theta_A \text{ and } e(p_m)=e_a=\overline{e}_a-\frac{\Delta\alpha}{\Delta c}\Delta\theta, \tag{2.64}$$

where $m=1,2$. In summary, if not a single non-affected firm opts in, then the prices and quantities are similar to our earlier formulas under imperfect participation without the voluntary provision.

2.3.6 Prices Versus Quantities

2.3.6.1 Comparative Advantage

We move to our main subject of this chapter, instrument choice under uncertainty. We introduced the concept of inefficient subsidization above. The agency takes the subsidization and the scope of the policy as given, so it regards these as the constraints in its optimization. Hence, in the study of the instrument choice, we not only concentrate on efficient outcomes but also study inefficient cases. We essentially review a continuum of effects that the different levels of subsidies—and consequently, the different levels of voluntary participation—produce. To accomplish this, the concept of comparative advantage will be applied. Remember that we already introduced this concept in Equation (2.22).

We study the choice between tax and permit system when either the brown technology in sector NA participates ($m=2$) or does not participate ($m=1$). We state

Proposition 4 *The comparative advantage between prices and quantities under voluntary participation is given by*

$$\Delta(\tau_m,p_m)=\frac{Z^*}{2}\rho_m\frac{Var(p_a)}{\gamma_a^2}(\gamma_a-\Theta_m\frac{n}{\rho_m}d), \tag{2.65}$$

where $Z^*>0$ *and* $m=1,2$. *The influence of voluntary participation on the instrument choice is given by* $\Theta_m\frac{n}{\rho_m}$, *where* $\Theta_m=\Theta(\tau_m,p_m)$ *is the volume effect,* n *is the scope effect, and* ρ_m *is the cost effect.*

As for the proof of the proposition, we refer to the analysis of an upcoming section (Section 2.3.7.1). Specifically, Equation (2.79) allows[24] us to write

$$\Delta(\tau_m, p_m) = \frac{\rho_m}{2} \frac{Var(p_m)}{\left(\gamma_m^L\right)^2} \left(\gamma - \frac{\Theta_m}{\rho_m} d\right), \tag{2.66}$$

where γ is the slope of the marginal abatement function. Furthermore, using the definitions of p (Equation (2.21)) and p_m (Equation (2.56)), we may write

$$\frac{Var(p_m)}{\left(\gamma_m^L\right)^2} = \left(\frac{1+uk_m}{1+ua}\right)^2 \frac{Var(p)}{(\gamma)^2},$$

where u, k_m, and a are defined in Equation (2.48) and $m = 1, 2$. Thus,

$$Z^* = Z\left(\frac{1+uk_m}{1+ua}\right)^2 > 0,$$

where Z is given by Equation (2.34). Finally, in writing Representation (2.65), we utilize our earlier discussion about the change in participation. In particular, we utilize Representation (2.33).

2.3.6.2 Combined Cost–Scope Effect

By Equation (2.65), the choice between the instruments τ_m and p_m is affected by the factor $\frac{n}{\rho_m}$. The term $n > 1$ is labeled the scope effect (Equation (2.28)). This effect arises when sector NA becomes incorporated into the pool of affected sectors. In the present context, scope effect records the effect that the switch from one efficient implementation to another creates in the instrument choice. Specifically, it records the effect that the reduction of the slope of the marginal benefit function creates.

We label the term ρ_m a cost effect. The cost effect supplements the scope effect by the fact that regulation is inefficient. Even though the scope of the regulation becomes wider, it inevitably becomes inefficient as well. Utilizing the variables k_m, u, and a (Equations (2.48)), we may write

$$\rho_m = \frac{\left(\gamma_m^L\right)^2}{\gamma\gamma_m^l} = 1 + \frac{u\,(k_m - a)^2}{(1+uak_m)^2}, \tag{2.67}$$

[24] It becomes clear that $v = v(\tau_m, p_m) = 0$ in Equation (2.66).

where $m = 1,2$. The cost effect disappears when $\rho_m = 1$. This occurs as the permit allocation induces efficient allocation of emissions ($k_m = a$). Under inefficient voluntary participation ($k_m \neq a$) instead, the cost effect invariably hurts the quantity instrument. In fact, we will later show that the cost effect in general favors the instrument with the lower variance. In each case, we see that the cost effect will increase as we move further away from the efficient solution.

Concerning the combined effect $\frac{n}{\rho_m}$, we argued above that the result $\frac{n}{\rho_m} > 1$ should hold in a meaningful policy. This condition guarantees that societal benefits exceed costs in the implementation of voluntary participation. By this argument, the combined cost–scope effect favors the quantity instrument.

2.3.6.3 Volume Effect

According to Section 2.3.7.1, we may write

$$\Theta_m = \Theta(\tau_m, p_m) = 2\frac{R_0(p_m)\left(\frac{\alpha_A}{c_A}\right)^2 + R_1(p_m)\left(\frac{\Delta\alpha}{\Delta c}\right)^2}{\left[R_0(p_m)\left(\frac{\alpha_A}{c_A}\right)\right]^2 + \left[R_1(p_m)\left(\frac{\Delta\alpha}{\Delta c}\right)\right]^2} - 1, \tag{2.68}$$

where $m = 1,2$. The factor Θ_m is called the volume effect. It gives the additional influence that the fluctuating quota induces on instrument choice. By the comparative advantage in Equation (2.65), whenever $\Theta_m > 1$, then the quantity instrument is favored. If $\Theta_m < 1$, the price instrument has an additional advantage. When $\Theta_m = 1$, the volume effect disappears. As for the particular size of the volume effect, denote first

$$q_m = q(p_m) = \frac{R_0(p_m)\left(\frac{\alpha_A}{c_A}\right)^2 + R_1(p_m)\left(\frac{\Delta\alpha}{\Delta c}\right)^2}{\left[R_0(p_m)\left(\frac{\alpha_A}{c_A}\right)\right]^2 + \left[R_1(p_m)\left(\frac{\Delta\alpha}{\Delta c}\right)\right]^2}. \tag{2.69}$$

Then, $\Theta(\tau_m, p_m) \gtreqless 1$, as $q_m \gtreqless 1$. By Equations (2.62), we may further write

$$q_m = \frac{1 + uk_m^2}{1 + uk_m a}\,\frac{1 + u^2 k_m a}{1 + u^2 k_m^2}, \tag{2.70}$$

where the factors $k_m \geq 0$, $u > 0$, and $a > 0$ are given in Equations (2.48). By straightforward calculations, we may state that

$$q_m \gtreqless 1 \Leftrightarrow k_m u(u-1)(a - k_m) \gtreqless 0. \tag{2.71}$$

In general, the volume effect is seen to vanish ($\Theta = 1$) if voluntary participation equals zero ($k_m = 0$) or is perfect ($k_m = a$). The effect also vanishes if the cost parameters are equal ($u = 1$).[25] We discuss above (see Equation (2.49)) how condition $a - k_m > 0$ is sufficient for a meaningful implementation of voluntary participation. However, by Equation (2.71), this condition alone does not favor a particular instrument. Rather, the volume effect favors the quantity instrument as long as $u > 1$. We also notice the possibility that $\Theta < 0$. Such an extreme outcome means that the tax instrument is a unanimously preferred instrument, so it should always be applied instead of instrument p_m. When compared to the traditional Weitzman framework, this is a new state of the world. It becomes possible solely because of the variability in the emissions under the quantity system. In particular, this effect is seen to arise as long as $q_m < \frac{1}{2}$.

2.3.6.4 Discussion of the Comparative Advantage

We just derived a comparison between subsidized prices and quantities. The analysis revealed that the choice of the instrument depends on the sign of the difference

$$\gamma_a - \Theta_m \frac{n}{\rho_m} d,$$

where γ_a and d are the slopes of the marginal benefit and damage functions, respectively, while Θ_m is the volume effect and $\frac{n}{\rho_m}$ is the combined cost–scope effect. A positive difference means that the tax is preferred, while negative values mean that the chosen instrument is quantities.

We discuss in Chapter 1 that the volume effect equals zero whenever the environmental agency controls the level of aggregate emissions. However, the emission control is only thinkable if the whole sector NA voluntarily participates ($m = 2$). The policy analysis in this case resembles the corresponding analysis in Chapter 1. If the sterilized system is used, then volume effect disappears ($\Theta_m = 1$), so the combined cost–scope effect will dominate. If only green technology participates ($m = 1$), then the emission control is impossible. The agency can control the total number

[25] If $u = 0$, we have $q_m = 1$. If $u \longrightarrow \infty$, it holds that

$$q_m = \frac{\frac{1}{u} + k_m^2}{\frac{1}{u} + k_m a} \frac{\frac{1}{u^2} + k_m a}{\frac{1}{u^2} + k_m^2} \underset{u \longrightarrow \infty}{=} \frac{k_m^2}{k_m a} \frac{k_m a}{k_m^2} = 1$$

by the use of Equation (2.69). In both cases, the volume effect vanishes.

of permits, but the number of permits does not equal the level of emissions. It can be shown that the nature of the instrument choice changes radically if the agency commits to the number of permits in a market, where total emission control is impossible. Although possible, we do not consider such a policy relevant, so we do not report the analysis here.

We present an extensive discussion about the influences of subsidization in Chapter 1. However, the analysis in there deals only with mandatory participation. The sectors in there are also more standard because no switch to green investment is possible. As for the imputed policy in there, the agency does not use supramarginal subsidization but rather applies inframarginal thresholds. Furthermore, the uncertainty is industry-wide, so every polluting unit in the polluting industry faces the same shock.

Despite these differences, the analysis in this section is very similar to that of Chapter 1. We comment on two major differences. First, the discussion about the scope effect is missing in Chapter 1 simply because the scope effect is solely due to imperfect participation. Second, the different assumptions about the nature of uncertainty produce some different policy implications between the studies. The uncertainty variables are identical in Chapter 1, while they are identically distributed here. The volume effects in particular reflect these differences. For example, the central role of the parameter u here is solely due to the assumed nature of uncertainty. We can show (see below) that this effect is truly due to the nature of the assumed uncertainty, not because of voluntary participation.

We take imperfect participation without voluntary participation as a point of departure. This means that any change between the instruments occurs relative to this regime. This choice implies that we are primarily interested in the expansion of the scope: The implementation of the voluntary provision means that the scope of the regulation increases. However, if one compares the changes in the instrument choice relative to the original Weitzman result (with perfect participation), then it will be appropriate to use the representation in Equation (2.66).

Overall, if $\Theta_m \frac{n}{\rho_m} > 1$, the relative position of the quantities is improved, while the value $\Theta_m \frac{n}{\rho_m} < 1$ indicates a stronger case for prices. We will calculate the size of the total effect in Appendix B.3. We show that the various individual effects will pull into opposite directions in such a way that no simple recommendations cannot be given. However, if we operate relative to the original Weitzman analysis (so that

$\frac{\Theta_m}{\rho_m}$ is the total effect), then more structure evolves (see Appendix B.3).

Finally, we note how the critical points $k_m = a$ (efficient implementation) and $k_m = 0$ (zero participation) are special as they both produce the traditional Weitzman measure. To see this, write first

$$\frac{Var\,(p_m)}{\left(\gamma_m^L\right)^2} = \frac{1+uak_m}{1+ua^2}\left(\frac{\alpha_A}{c_A}\theta_A - \frac{k_m}{a}\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2,$$

where $m = 1,2$. The outcome $k_m = a$ implies that $\Theta_m = 1$, $\rho_m = 1$, and that

$$\frac{Var\,(p_m)}{\left(\gamma_m^L\right)^2} = \left(\frac{\alpha_A}{c_A}\theta_A - \frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2 = \frac{Var(p)}{\gamma^2}.$$

Thus, by Equation (2.66),

$$\Delta(\tau_m, p_m) = \frac{\rho_m}{2}\frac{Var\,(p_m)}{\left(\gamma_m^L\right)^2}\left(\gamma - \frac{\Theta_m}{\rho_m}d\right) \underset{k_m=a}{=} \frac{1}{2}\frac{Var(p)}{\gamma^2}(\gamma - d), \tag{2.72}$$

so $\Delta(\tau_m, p_m) = \Delta(\tau, p)$, where $m = 1,2$.

Second, the value $k_m = 0$ implies that $\Theta_m = 1$. Furthermore,

$$\rho_m = 1 + \frac{u\,(k_m - a)^2}{(1+uak_m)^2} \underset{k_m=0}{=} 1 + ua^2 = n.$$

So,

$$\Delta(\tau_m, p_m) \underset{k_m=0}{=} \frac{n}{2}E\left[\frac{\alpha}{c}\theta_A\right]^2(\gamma - \frac{1}{n}d), \tag{2.73}$$

or after applying the relation $\gamma_a = n\gamma$,

$$\Delta(\tau_m, p_m) = E\left[\frac{\alpha}{c}\theta_A\right]^2(\gamma_A - d) = \Delta(\tau_a, p_a),$$

where $m = 1,2$. In summary, the critical points will yield the traditional Weitzman measures. This will happen because the implementations are efficient at these points. It also happens that both the cost and volume effects disappear at the critical points. This holds if only green technology participates ($m = 1$) or if the whole sector NA participates ($m = 2$). Note also the common source behind the cost and volume effects. They both require inefficient implementation to operate.

2.3.7 Further Issues in Comparative Advantage

2.3.7.1 The Derivation of Comparative Advantage

Recall that instruments operate in one of the two mutually exclusive regimes m so that $m = 1,2$. A unifying theme across different implementations is the certain structure that various prices and quantities follow. According to Section 2.3.5.2, we may write every price as

$$s_m = \bar{s}_m + \gamma_m^L \left(R_0(s_m) \frac{\alpha_A}{c_A} \theta_A + R_1(s_m) \frac{\Delta\alpha}{\Delta c} \Delta\theta \right)$$

and every quantity as

$$e(s_m) = \bar{e}_m + (1 - R_0(s_m)) \frac{\alpha_A}{c_A} \theta_A + (1 - R_1(s_m)) \frac{\Delta\alpha}{\Delta c} \Delta\theta,$$

where $R_0(s_m)$ and $R_1(s_m)$ are the implementation-specific parameters, and $E s_m = \bar{s}_m$ and $E e(s_m) = \bar{e}_m$. Furthermore, referring to Equation (B.12) (Appendix B.1), we can write the social welfare as

$$B(s_m) - D(e(s_m)) = B_U - \frac{1}{2\gamma_m^l} s_m^2 - \frac{d}{2} (e(s_m))^2, \tag{2.74}$$

where B_U are the counterfactual benefits.

Consider the comparative advantage $\Delta(I,J)$ between the two instruments I and J as defined in Equation (2.22). We apply the welfare in Equation (2.74) when we write the comparative advantage in terms of variances[26] as

$$\Delta(I,J) = \frac{Var(J) - Var(I)}{2\gamma_m^l} - \frac{d}{2} (Var(e(I) - Var(e(J))).$$

Expanding the different variances gives us

$$\Delta(I,J) = Var(J) \frac{1-v}{2} \left[\frac{1}{\gamma_m^l} - d \frac{1}{(\gamma_m^L)^2} \left(2q(J) \frac{1 - v\frac{q(I)}{q(J)}}{1-v} - 1 \right) \right], \tag{2.75}$$

[26] We also apply calculations in Appendix A.6.

where

$$q(J) = \frac{R_0(J)\left(\frac{\alpha_A}{c_A}\right)^2 + R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\right)^2}{\left[R_0(J)\left(\frac{\alpha_A}{c_A}\right)\right]^2 + \left[R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\right)\right]^2} \tag{2.76}$$

and

$$v = v(I,J) = \frac{Var(I)}{Var(J)}.$$

The representation in Equation (2.75) is by no means obvious, so we will present the derivation of it in Appendix B.4. We further modify the comparative advantage to yield

$$\Delta(I,J) = \frac{\rho_m (1-v)}{2} \frac{Var(J)}{\left(\gamma_m^L\right)^2}\left(\gamma - d\frac{1}{\rho_m}(2Q-1)\right), \tag{2.77}$$

where

$$Q \equiv Q(I,J) = q(J)\frac{1 - v\frac{q_m(I)}{q_m(J)}}{1-v} \tag{2.78}$$

and ρ_m is given by Equation (2.67). Finally, we denote the volume effect by Θ_m, so that

$$\Delta(I,J) = \frac{\rho_m (1-v)}{2} \frac{Var(J)}{\left(\gamma_m^L\right)^2}\left(\gamma - d\frac{\Theta_m}{\rho_m}\right), \tag{2.79}$$

where

$$\Theta_m \equiv \Theta_m(I,J) = 2Q(I,J) - 1. \tag{2.80}$$

We would like to comment on these measures in relation to our earlier representations in Chapter 1. In both chapters, we derive the instrument choice in terms of various variances. In the present case, it holds that

$$Var(I) = \left(\gamma_m^L\right)^2 \sigma^2 \left[\left(R_0(I)\frac{\alpha_A}{c_A}\right)^2 + \left(R_1(I)\frac{\Delta\alpha}{\Delta c}\right)^2\right] \tag{2.81}$$

and

$$Var(e(I)) = \sigma^2\left[\left((1-R_0(I))\frac{\alpha_A}{c_A}\right)^2 + \left((1-R_1(I))\frac{\Delta\alpha}{\Delta c}\right)^2\right]. \tag{2.82}$$

for instrument I (see Section 2.3.5.2). Compare these to our earlier representations of Equations (1.59) and (1.60) in Chapter 1. The formulas have much in common but they are different. The sole reason behind the differences lies in the different natures of uncertainty. In Chapter 1, the stochastic variables are identical across the polluting units, while they are identically distributed here. The analysis of instrument choice reflects the difference as well. The dominant role of factor u (see Equation (2.71)) displays this fact.

Second, there is an important and illuminative representation between $Var(I)$ and $Var(e(I))$. We calculate in Appendix B.5 that

$$Var(e(I)) = Var(U) - Var(I)\frac{2q(I)-1}{\left(\gamma_m^L\right)^2}. \tag{2.83}$$

As $Var(U)$ is the variance of emissions in the absence of regulation, the representation shows how the environmental policy succeeds to change it. Thus, as long as $q(I) > \frac{1}{2}$, policy I will succeed in reducing the variance in emissions. Based on Equation (2.68), we may also write

$$Var(e(I)) = Var(U) - Var(I)\frac{\Theta(\tau_m, I)}{\left(\gamma_m^L\right)^2}.$$

Specifically, we have $Var(e(\tau_m)) = Var(U)$.

2.3.7.2 Quantities Versus Quantities

We make a brief note about "quantities versus quantities" here. We discuss the issue in Chapter 1 at length in a perfect participation framework. In the present context, the issue is valid, if the whole sector NA voluntarily participates. In that case, the total emission control is feasible.

In brief, two different quantity instruments evolve as the agency may either commit to the total number of permits or to the total number of auctioned permits, respectively. Given that every firm in the polluting industry is regulated, the first option fixes the aggregate emission level while the second does not. Assume that the policy indeed fixes the emissions and let us denote the corresponding permit price in the market by p_2^L. By the representation in Equation (2.77), the comparative advantage is

$$\Delta(p_2^l, p_2^L) = \frac{\rho_2(1-v)}{2}\frac{Var(J)}{\left(\gamma_2^L\right)^2}\left(\gamma - \frac{d}{\rho_2}(2Q-1)\right),$$

where $v = v(p_2^l, p_2^L)$ and $Q = (p_2^l, p_2^L)$. It can be shown that

$$R_0(p_2^L) = R_1(p_2^L) = 1,$$

so (by Equation (2.76)),

$$q_2(p_2^L) = 1$$

and (by Equation (2.78)),

$$Q(p_2^l, p_2^L) = \frac{1 - vq(p_2^l)}{1-v}.$$

The term $q(p_2^l)$ was earlier calculated in terms of k, a and u in Equation (2.70). To study the size of factor $Q(p_2^l, p_2^L)$ further, we need to know more about the variance factor v.[27]

2.3.7.3 Perfect Participation: Instrument Choice

The subsidization has a socially beneficial role above as it supplements incomplete environmental policy. We will briefly discuss next how the subsidization framework adapts to a more standard setting, namely, to the regulation of an entire polluting industry. Thus, we will return to the main theme of Chapter 1. There are two main differences between the chapters. First, we do not assume here that uncertainty variables are identical but rather that they are identically distributed. Second, the motive

[27] Based on intuition, we are inclined to predict that $Var(p_2^L) > Var(p_2^l)$, so that

$$0 < v = \frac{Var(p_2^l)}{Var(p_2^L)} < 1.$$

After all, the non-sterilized system does not limit the emissions level to a fixed quantity but allows it to accommodate. Lower variance in price should then reflect this accommodation. However, this intuition is incorrect. It can be shown that the variance does not invariably decrease if we let the emissions fluctuate.

for subsidization differs between chapters. The motive here is not about supporting the polluting firms against closures but rather to promote the transformation towards green production.

Let us then assume that the aim of the policy under perfect participation is to provide an extra boost to green technology. Consequently, the aim of the subsidization is the same as in the main text above, whereby increasing the participation means in practice that investments in green technology should be supported. The major change is that we no longer have to apply supramarginal thresholds, but we may use inframarginal thresholds as well. In mathematical terms, increasing green production when every firm participates means that $\Delta\alpha - \Delta l \geq 0$ (see Conditions (2.10) and (2.41)). We can deduce that this target can be implemented with various thresholds. In particular, it is implementable by supramarginal policy ($l_1 > \alpha_1$) or by inframarginal policy ($\alpha_1 > l_1$).

Adaptation to perfect participation is fairly simple. We just apply the framework that was originally written to the policy where whole sector NA voluntarily participates.[28] Consequently, the comparative advantage derived in Equation (2.79) is useful as such in the study of perfect participation. In particular, we get the cost effect ρ_2 and the volume effect Θ_2 as we insert

$$k_2 \equiv \frac{\Delta\alpha - \Delta l}{\alpha_A} > 0$$

into Equations (2.67) and (2.68), respectively. By Equation (2.71), the volume effect will favor the quantity instrument (so that $\Theta(\tau_2, p_2) > 1$) as long as

$$k_2 = \frac{\Delta\alpha - \Delta l}{\alpha_A} > 0$$

where $a - k_2 = \frac{\Delta l}{\alpha_A}$. Assume that the policy subsidizes green technology ($l_1 > 0$) and does not subsidize brown technology ($l_0 = 0$). This means that $\Delta l = -l_1 < 0$, so $a - k_2 < 0$. Thus, the volume effect will favor the quantity instrument as long as $u < 1$. In other words, it will favor the quantity instrument as long as $\Delta c > c_A$.

Finally, let us briefly comment on the assumption $l_A = 0$ that we have applied so far. By construction, the influence of l_A in the instrument choice is channeled through factor k_2. We may then conclude that setting $l_A > 0$ merely affects the value

[28] This means that one has to apply values $\iota_1 = 1$ and $\iota_2 = 1$ everywhere (see Equations (2.52) and (2.53)).

of k_2. Written in full, we have

$$k_2 = \frac{\Delta\alpha - \Delta l}{\alpha_A - l_A}.$$

2.3.7.4 Perfect Participation: To Subsidize or Not to Subsidize

Our analysis has focused on the subsidization of green technology in a regulation whereby market-based instruments regulate the emissions. Furthermore, our policy-making is restricted by inefficient subsidization. If the environmental agency alone could decide, it would promote neutrality. In the present context (with perfect participation), the neutrality condition can be found from Equation (2.37). Accordingly, if the brown production in sector A (all firms in there use only brown technology) is not subsidized at all ($l_A = 0$), then the neutrality principle rules that green technology should not be favored at the expense of brown technology. In other words, the principle advises to use the rule $l_1 = l_0$.

We briefly consider another case of subsidization under perfect participation. We show how the subsidization of green technology is good policy when this policy is the only policy available. In a sense, this approach represents another example of restricted policy implementation.

Let the subsidy equal S. It is paid to firms applying green technology. As the subsidy is the only instrument in the regulation, it affects only units in sector N. Specifically, the number of firms in sector N becomes (Equation (2.9))

$$\lambda_1 = \frac{\Delta b - \Delta\theta + S}{\Delta c}.$$

We are interested in the change of the expected social welfare that the introduction of subsidization induces. We have[29]

$$\frac{dEW}{dS} = E\left[\left((B_1(\lambda_1) - B_0(\lambda_1)) - D'(e)\frac{de}{d\lambda_1}\right)\frac{d\lambda_1}{dS}\right],$$

where

$$\frac{d\lambda_1}{dS} = \frac{1}{\Delta c} > 0$$

[29]The various components in welfare are introduced in Section 2.2.1.

and (by Equation (2.13))

$$\frac{de}{d\lambda_1} = -\Delta\alpha < 0.$$

Furthermore, the discussion about the polluting industry in Section 2.2.1 implies that

$$B_1(\lambda_1) - B_0(\lambda_1) = -S.$$

Thus, we have

$$\frac{dEW}{de} = \left(-S + ED'(e)\Delta\alpha\right)\frac{1}{\Delta c}.$$

We are specifically interested in the change in social welfare that the introduction of subsidization induces. We have

$$\left.\frac{dW}{de}\right|_{S=0} = \frac{\Delta\alpha}{\Delta c} E\left[D'(U)\right],$$

where U is the level of counterfactual emissions (Equation (2.19)). As $D'(e) > 0$, then

$$\left.\frac{dW}{de}\right|_{S=0} > 0.$$

We may conclude that the introduction of subsidization is welfare enhancing.

2.4 Concluding Remarks

Concerning questions of voluntary participation, the theoretical literature has focused on the design of tradable permit systems. We supplement this literature by taking a step back and asking whether environmental taxes can outperform tradable permits. We employ environmental taxation as an alternative and equal means of dealing with the policy questions concerning imperfect participation and provision of voluntary participation. This is possible because, in principle, taxes and tradable permits share the same distributional properties. We find that voluntary participation does affect the instrument choice as long as the policy applies inefficient implementation.

Fundamentally, in thinking about the state of the regulation, participation is either perfect or imperfect. Within imperfect participation, provision of voluntary participation aims to increase the participation rate toward perfect participation. We study voluntary participation in a framework where participation and green investments are linked to each other. Firms will make new green investments because of the incentives that voluntary participation provides to them. However, we find a fundamental trade-off that governs the voluntary provision. The provision is certainly beneficial as it widens the scope of the regulation towards profitable environmental projects. At the same time, it attracts these projects with subsidies that turn the emission allocation inefficient. We start our analysis by identifying socially beneficial subsidization profiles. After that, we compare how prices and quantities perform in implementing these beneficial voluntary provisions.

Overall, our focus is on subsidization that supports investments towards greener production. We build a framework that allows the analysis under both perfect and imperfect participations. We identify three central influences in the instrument choice: scope, cost and volume effects. The scope effect records the influence that the increased scope of regulation has on instrument choice. The term “increased scope" means that not every polluting firm is regulated originally but that the number will increase thanks to environmental program. Consequently, this effect is present only in cases that include imperfect participation. The scope effect basically records the consequences of the policy to the benefit side. Specifically, the increased scope reduces the slope of the marginal benefit function. In instrument choice, this is a beneficial change to the quantity instrument. We conclude in the main text that the quantity instrument becomes more attractive if the environmental agency succeeds in increasing participation in an efficient manner. Cost effect is a consequence of an inefficient allocation. To enable the increased scope of the regulation, the policy must necessarily apply subsidies, and this opens the door for inefficiencies. We show that, unlike the scope effect, the cost effect invariably favors the price instrument. The volume effect adds to the instrument choice the fact that emissions will fluctuate under the quantity instrument. As in Chapter 1, this may or may not benefit the quantity instrument. The cost and volume effects work both in imperfect and perfect participation regimes.

We already described the workings of cost and volume effects in Chapter 1. The special issue in the current chapter concern these effects when only a fraction of firms participate voluntarily. Eventually, there may still exist polluting units in the polluting industry that will lie outside the regulation. One feature in our framework is that the traditional rules in instrument choice apply as long as voluntary provision is implemented efficiently.[30] We emphasize that this is the case even though there are polluting units lying outside the regulation. If voluntary provision is implemented inefficiently, cost and volume effects will arise and the instrument choice is affected. Based on our model, we may conclude that voluntary participation in itself does not affect the instrument choice but that inefficiency in the implementation does.

We motivate this chapter by voluntary participation under imperfect participation, but we discuss how the analysis is valid in the cases of perfect participation as well. Furthermore, even though the policy supports green investments by supramarginal thresholds, the analysis is valid in cases of inframarginal thresholds as well. We only require that the subsidization policy should encourage the application of green technology in every implementation. More formally, we show in the main text that this target requires $\Delta\alpha - \Delta l \geq 0$, where $\Delta\alpha = \alpha_0 - \alpha_1 > 0$ is the difference between brown and green emissions and $\Delta l = l_0 - l_1$ is the difference between brown and green emission thresholds. As $\Delta\alpha - \Delta l = (\alpha_0 - l_0) - (\alpha_1 - l_1)$, the targeted policy is implementable either by supramarginal policy ($l_1 > \alpha_1$) or by inframarginal policy ($\alpha_1 > l_1$). The reader may also find that the volume effect is somewhat elaborated in this chapter. This is because we do not assume that uncertainty variables are identical but identically distributed.

In deriving these new results, our work mixes three distinct branches of literature: one concerning long-run efficiency properties of taxes and tradable permits (Farrow [18]; Pezzey [54]), the second concerning voluntary participation (Montero [47]), and the third concerning inefficient policy implementation under uncertainty (Chapter 1; Montero [49]). The novel framework urges new applications that provide new intuition.

[30] The efficiency issue is similar to Montero [47]. However, in his model, efficient voluntary implementation corresponds to a market where every polluting unit participates.

3 MULTIPLE EXTERNALITIES

3.1 Some Background

One general ambition of environmental policy-making is the transition towards environment-friendly green technology. Within this approach, green investments replace brown polluting thereby reducing the amount of negative environmental externalities (pollution). One way to support this transition is to make the brown technology relatively more expensive. In theory, this may occur either by subsidizing green technology or by taxing the use of brown technology. However, internal forces of the polluting industry may also affect the speed of transition from brown to green technology. By these forces, we mean the various interdependencies between firms. One such interdependency is that additional application of a certain technology becomes easier as the number of appliers increases. Such effects are external to individual firms, so they can be labeled as technological externalities (Griliches [21], [22]). Thus, if a positive technological externality exists and if the direction of this effect is clear, then the transition towards green technology can be enhanced by supporting the externality-generating sector. Therefore, we have to take into account two externalities: the environmental externality, which is external, and the technological externality, which is internal to the polluting industry. (Jaffe, Newell, & Stavins [25].)

Our prime question concerns the instrument choice under uncertainty. We note from the outset that the instruments in this chapter are not standard. They include simultaneous regulation of two separate externalities. On one hand, the choice is made between market-based instruments (i.e., tradable permits and environmental taxes) to deal with the environmental externality, that is the homogenous pollution. On the other hand, there exists another externality: the technological externality. This means that the additional use of a certain production technology in one sector has some future effects on the productivity in the other sector. Our first question concerns the necessity of an additional regulation in dealing with the technological externality. Second, if the policy needs additional regulation, there is a further question about the implementation of the policy, namely, how does the price and quantity instruments compare when there is more than one externality concerned.

In our model, two sectors constitute a polluting industry. In both sectors, firms have access to two technologies: brown and green. The industry inherits brown technology from the past, when pollution was not a big issue in private investment calculations. Green technology is a new technology that will gradually replace brown technology. Among other benefits, green technology has a lower environmental burden than brown technology. Of the various possible interdependencies between different industries and technologies, we will pick one case for closer review. We examine a sector that plays a leading role in the introduction of green technology. The accumulated experiences in this leading sector affect the introduction of green technology in the other sector. Likely, the more the firms in the leading sector switch from brown to green technology, the easier it will be for the firms in the other sector (the follower) to do the same.[1]

We argue that the policy should internalize the externalities by economic incentives. In the present context, this means that various payments should be applied. For example, the use of technology standards (including the use of a uniform technology standard) is a potential tool in the policy mix. In our model, however, the technology standards approach does not work properly. The efficient allocation of emissions requires simultaneous use of both green and brown technology. The policy-making is complicated by the fact that the agency lacks data of firm-specific efficiencies in utilizing different technologies. Together, these two things imply that technology standards cannot replace economic incentives.

As for the content of the payment policy, we divide a payment into two parts. The first part, the environmental payment, is the price of emissions times the amount of emissions. Every polluting unit must make this payment. The second part is a subsidy payment that is paid to a polluting unit that generates a positive spillover. We show that this positive subsidy payment to a polluter is an essential part of optimal policy.

Our results bring new insights into the implementation issues of environmental policy. Whenever there exist several externalities, the results help in selecting the

[1]We will follow this view most of the time by assuming a strictly positive externality throughout the work. We will discuss the negative case briefly at the end of the work.

proper strictness of the policy as well as in choosing between various instruments in the implementation of the policy. In general, the study contributes to the issue of firm subsidization. We show how the environmental payment alone does not yield social optimality. We give an efficiency-based reason for subsidies to internalize the technological externalities. We are then studying a regime ("the efficient regime") *where the policy applies subsidies for reasons of efficiency*. Furthermore, as a secondary objective, we study a more restricted framework. We do not take for granted that the environmental agency always has the authority to internalize both the negative environmental externality and the positive technological externality. This observation leads us to study a policy in which the regulatory agency controls only the negative environmental externality. In other words, we also study a regime ("the inefficient regime") *where the policy does not apply subsidies, even though it should for efficiency reasons.*

There is a further dimension in policy-making, as the agency may choose between fixed and fluctuating payments. The regulation is organized by market-based instrumentation. This means that the regulatory choice is between pollution taxes and tradable permits. The payment structures are designed to include the unit price of emissions (tax rate or permit price). The price is further designed to affect both parts of the payment, the environmental payment and the subsidy. Note also that we assume uncertain benefits in the polluting industry. With these things taken together, the tax implementation means fixed payments while the permit implementation means fluctuating payments.

We end up studying instrument choice between subsidized prices and quantities. We find two new effects in the instrument choice (cf. Weitzman [84]). The slope effect captures the fact that the spillover effect makes the abatement curve less steep.[2] This feature will invariably favor the quantity instrument. We also find other effects that we collect under the term "cost effect." In the previous chapters, we showed how the cost effect favors the price instrument. In the implementations of this chapter, the cost effect favors the price instrument as well. However, unlike in the earlier chapters, here the cost effect merely reflects *ex-post* inefficiency. The polluting sectors become firmly connected by the spillover effect. Even though the environmental agency deliberately develops efficiency *ex-ante,* the policy does not remain efficient *ex-post*. In every case, we show that the various parameters of the model

[2]The slope effect is a base effect discussed in the Introduction. The scope effect in Chapter 2 is another base effect.

determine the size of the cost effect. The strength of the spillover effect is one such parameter.

Chapters 1 and 2 provide earlier examples of the relationship between the environmental payment and the subsidy. In there, the central theme is the study of instrument choice when the policy must apply inefficient subsidization. In Chapter 1, we give no explicit reason for the use of subsidization. One plausible explanation relates to the influence of various stakeholders and interest groups. After all, subsidization can be used for various purposes, such as lowering the environmental payments of polluting firms or as boosting green investments. Conversely, Chapter 2 offers a welfare-enhancing reason for subsidies. The policy employs subsidies in a program of voluntary participation, and regulation expands to include so-called non-affected firms in this program. We discuss how the first-best policy becomes easily inaccessible in such a program, either because of technical or political restrictions (see also Montero [49]).

Meunier [43] discusses instrument choice in an inefficient regime. He argues that his model can be applied in the case of simultaneous regulation of environmental externalities and knowledge spillovers.[3] The Meunier model and our model share the linear-quadratic framework, but they differ in some important respects. Meunier describes the efficient policy only briefly, while our work emphasizes subsidization and the variety of tools that the efficient implementations can apply.[4] Second, our work assumes that the polluting industry (i.e., the pool consisting of every polluting unit) is under the control of the regulatory agency. In Meunier, instead, one of the industries remains beyond regulation.[5]

Requate [61] discusses the link between environmental policy instruments and advanced environmental technology. Jaffe, Newell, and Stavins [25] discuss the simultaneous presence of knowledge spillover and an environmental externality. The literature also includes earlier models that mix the choices of environmental policy

[3]For further discussion about the knowledge spillovers in this particular context, see the references in Meunier [43]. Heal and Tarui [23] and Smulders and DiMaria [70] offer a supplementary look at the subject.

[4]In addition to knowledge spillovers, we can find other types of externalities that promote efficient subsidization. For example, Shinkuma and Sugeta [68] study instrument choice in the long run, when the entry itself creates an externality. We discuss this study in the Introduction.

[5]In describing the efficient regime, Meunier [43] briefly refers to the existence of Pigouvian taxes. According to our study, the implementation in the efficient regime includes elements from taxation and subsidization. Overall, the regulatory regime in Meunier [43] differs significantly from ours, so the second-best results in Meunier do not directly apply to our study in this chapter.

instruments and environmental production technologies in a Weitzman [84] framework. These include D'Amato and Dijkstra [10], Krysiak [34], and Mendelsohn [42]. The model of D'Amato and Dijkstra [10] can be considered as a pure model of adoption.[6] Conversely, in Mendelsohn [42], there is an explicit (uncertain) relationship between technological advances and R&D expenses. Specifically, for Mendelsohn [42], innovation is a pure private good, so the firm under investigation has proper private incentives.[7] Innovation effectively makes the long-term marginal cost curve less steep than the corresponding short-term curve (with zero innovation). Then, as compared to a world with zero innovation, the quantity instrument is favored. This follows from the basic principles of instrument choice. Accordingly, as the relative importance of pollution damages increases, the relative importance of quantity control increases.

In our model, the slope effect captures practically the same effect that the Mendelsohn model does, namely, that investment will make the slope of the abatement function less steep. This favors the quantity instrument. However, in our model, there is an additional effect: the cost effect that reflects inefficient subsidization. As we argue above, the cost effect favors the price instrument, so it draws the instrument choice in the opposite direction. In summary, we will supplement the Mendelsohn model by the effects that inefficient subsidization creates. In the Mendelsohn model, all the gains of investments are purely private. There is no reason for subsidization and, correspondingly, no room for cost effects.

3.2 The Model

3.2.1 The Polluting Industry

The polluting industry consists of two sectors: g and r. A sector consists of numerous infinitesimally small firms. Within a sector, every firm may choose between two production technologies, 0 and 1, to produce the commodity unit. In sector k, the

[6] The adaptation in D'Amato and Dijkstra [10] means a discrete jump in the marginal function. Here, the adaptation is a smooth and continuous process.

[7] In other words, no technological externalities or knowledge spillovers exist. This implies that there is no efficiency-based reason to subsidize the investment.

benefit for unit λ^k after choosing technology i is

$$B_i^k(\lambda^k) = b_i^k + \theta_i^k - c_i^k \lambda^k + F_i^k, \tag{3.1}$$

where $i = 0, 1$ and $k = g, r$. The factors b_i^k and c_i^k are positive constants, and θ_i^k's are technology- and sector-specific random shocks. We assume that $\Delta b^k = b_1^k - b_0^k > 0$ and $\Delta c^k = c_1^k - c_0^k > 0$ and denote $\Delta\theta^k = \theta_0^k - \theta_1^k$, where $k = g, r$. Factor F_i^k is called an externality effect.

We assume that the uncertainty in our model contains an aggregate and an idiosyncratic part. More precisely, the total shock consists of a technology-specific shock (ϵ_i^k) and an economy-wide shock (ϵ). Every producer in the economy shares the economy-wide shock. Furthermore, the size of the technology-specific shock depends on the sector in which the technology is applied. The uncertainty takes an additive form, which means that

$$\theta_i^k = \frac{\epsilon + \epsilon_i^k}{2},$$

where $i = 0, 1$, and $k = g, r$. In particular,

$$\Delta\theta^k = \frac{\epsilon_0^k - \epsilon_1^k}{2},$$

so the aggregate shock is seen to cancel out from the variable $\Delta\theta^k$. It also holds that the variables ϵ and ϵ_i^k are identically and independently distributed random variables, so $E(\epsilon) = E(\epsilon_i^k) = 0$ and $Var(\epsilon) = Var(\epsilon_i^k) = \sigma^2$, with $i = 0, 1$ and $k = g, r$. Then, $E(\Delta\theta^k) = 0$ and $Var(\Delta\theta^k) = \sigma^2$, with $k = g, r$. We will further simplify our analysis as we assume only one externality effect. We let

$$F_0^g = F_1^g = F_0^r = 0$$

and

$$F_1^r = \phi \lambda_1^g > 0,$$

where λ_1^g is the number of firms that use technology one in sector g.

In principle, externalities can flow between and within the technologies and sectors and they can be either negative or positive.[8] However, we substantially restrict

[8] Appendix C.1 discusses a general linear externality effect.

the externality flows from the outset. There is only one effect within the whole polluting industry. The externality flows from sector g to sector r between units that use technology one. We then assume that sector g is an externality generator while sector r is an externality recipient. In what follows, we denote technology zero as brown (polluting) technology and technology one as green (clean) technology. This means that the externality exists within the green, not within the brown technology. Furthermore, the assumed positive effect implies positive spillovers: The entrant creates not only private benefits but also public benefits.[9] Overall, the application of green technology in sector g will increase the productivity of green technology in sector r. As always, the most important feature of an externality is that it causes real effects but it has no price.

3.2.2 The Externality-Generating Sector

In sector g, the benefit for unit λ^g after choosing technology i is

$$B_i^g(\lambda^g) = b_i^g + \theta_i^g - c_i^g \lambda^g, \tag{3.2}$$

where $i = 0, 1$. The environmental regulation means a policy implementation that applies either tradable permits or environmental taxes. We use s to denote the unit price of emissions. The profit for unit λ^g using technology i becomes

$$\Pi_i^g(\lambda^g) = B_i^g(\lambda^g) - s\alpha_i^g + S_i^g(s), \tag{3.3}$$

where $i = 0, 1$. The parameter α_i^g is the level of emissions if a unit utilizes technology i. The factor $S_i^g(s)$ is a technology- and sector-specific payment rule. Most importantly, it depends on the unit price of emissions. If $S_i^g(s) > 0$, it is a subsidy. It is then paid to an active firm producing in sector g and using technology i. As the policy regulates the production of emissions, the emission content of the production is important. We assume that $\Delta\alpha^g = \alpha_0^g - \alpha_1^g > 0$, so the emission level of producing a unit of commodity is lower using technology one than using technology zero. We take technology zero as an incumbent (polluting) technology, while technology one stands for new (clean) technology. Thus, we can consider the switch from technology zero to technology one as a green investment.

[9]The negative effect, in turn, implies that some scarce resources are congested. Product market rivalry may also explain the negative sign.

We assume that the size of the sector g is fixed and it equals λ_0^{g+}. Furthermore,

$$B_0^g(\lambda_0^{g+}) = b_0^g + \theta_0^g - c_0^g \lambda_0^{g+} > 0.$$

Within sector g, a cut-off unit exists that is indifferent between two options. The cut-off $\lambda_1^{g+}$is indifferent between using technology zero and one. It holds that $B_0^g(\lambda_1^{g+})$ $= B_1^g(\lambda_1^{g+})$, so

$$\lambda_1^{g+}(\theta) = \frac{\Delta b^g - \Delta\theta^g}{\Delta c^g}. \tag{3.4}$$

Note that λ_1^{g+} may well take values greater than zero, so the model includes the possibility that some firms use clean technology even in the absence of regulation.

The introduction of the regulation influences the determination of the cut-off unit. Denote the new unit by λ_1^g. We have

$$\lambda_1^g = \frac{\Delta b^g - \Delta\theta^g + s\Delta\alpha^g + \Delta S^g}{\Delta c^g} = \lambda_1^{g+} + \frac{\Delta\alpha^g}{\Delta c^g}s + \frac{\Delta S^g}{\Delta c^g}, \tag{3.5}$$

where $\Delta S^g = S_1^g - S_0^g$. Written this way, λ_1^g is a response function that depends on the policies s and ΔS^g. As far as $s > 0$, the condition $\Delta S^g > 0$ is a sufficient condition so that emissions are reduced by green investments.

Figure 3.1 illustrates the regulatory outcome in sector g. The line $B_i^g(\lambda^g)$ corresponds to sector i business-as-usual profits, while the line $\Pi_i^g(\lambda^g)$ involves the influence of the regulation, $i = 0, 1$. We use solid lines to describe the profits in the absence of regulation, while the non-solid lines depicts regulated profits. In particular, note how our assumptions about Δb^g and Δc^g determine the structure of the sector. These assumptions mean that firms at the low end of the distribution apply green technology, while firms at the high end use brown technology. Furthermore, we have assumed that $\frac{\Delta\alpha^g s + \Delta S^g}{\Delta c^g} > 0$, so the utilization of green technology increases. Types $\left[0, \lambda_1^g\right]$ use green technology while types $[\lambda_1^g, \lambda_0^{g+}]$ utilize polluting brown technology. As compared to the business-as-usual choices, types $\left[\lambda_1^{g+}, \lambda_1^g\right]$ switch from brown to green technology.[10]

In earlier chapters, the payment rules $(S_i^g(s))$ depend on the technology- and

[10]It is perfectly feasible that green technology profits increase because of regulation in sector g. In terms of Figure 3.1, the line $\Pi_1^g(\lambda^g)$ would shift outwards. Investments in green technology will clearly increase.

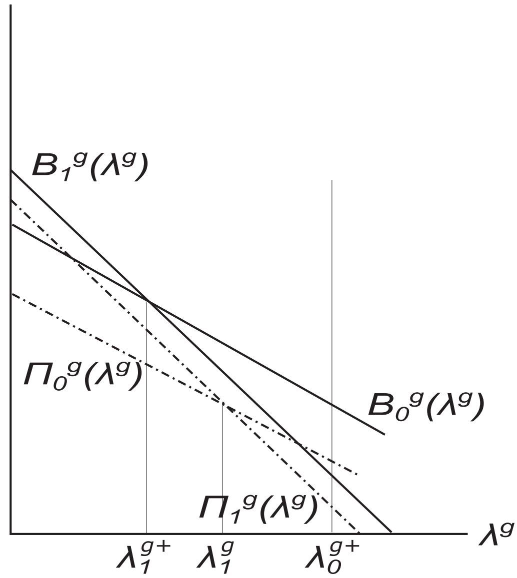

Figure 3.1 The Impact of Regulation in Sector g

sector-specific threshold levels (l_i^g). We can apply thresholds here as well. Let

$$S_i^g(s) = s l_i^g,$$

where $i = 0, 1$. We have

$$\Pi_0^g(\lambda_0^g) = \Pi_1^g(\lambda_1^g) \iff \lambda_1^g(\theta) = \lambda_1^{g+}(\theta) + \frac{(\Delta\alpha^g - \Delta l^g)}{\Delta c^g} s, \tag{3.6}$$

where $\Delta l^g \equiv l_0^g - l_1^g$. Thus, if the agency wants to reduce emissions through investments, it should set $\Delta\alpha^g > \Delta l^g$, or equivalently, set

$$\alpha_0^g - l_0^g > \alpha_1^g - l_1^g. \tag{3.7}$$

In the derivation above, we allow the possibility that the agency can apply payment rules that differ between technologies (i) and sectors (k). It could also be the case that the policy cannot apply technology-specific subsidies. That is, it can use only sector-specific subsidies. This assumption has immediate consequences. After inserting the new assumption $S_1^g = S_0^g$ into Equation (3.5), we have

$$\lambda_1^g = \lambda_1^{g+}(\theta) + \frac{\Delta\alpha^g}{\Delta c^g} s.$$

Thus, the agency is unable to promote green investments by this type of subsidization.

3.2.3 The Externality-Receiving Sector

In sector r (as in sector g), every firm chooses between technologies zero and one. Then, a unit λ^r has either

$$B_0^r(\lambda^r) = b_0^r + \theta_0^r - c_0^r \lambda^r \tag{3.8}$$

or

$$B_1^r(\lambda^r) = b_1^r + \theta_1^r - c_1^r \lambda^r + \phi \lambda_1^g \tag{3.9}$$

after choosing either technology zero or one, respectively. The corresponding profits are

$$\Pi_i^r(\lambda^r) = B_i^r(\lambda^r) - s\alpha_i^r + S_i^r(s), \tag{3.10}$$

where α_i^r is the emission content of technology i, while $S_i^r(s)$ is the payment rule, $i = 0, 1$. Factor $\phi\lambda_1^g$ is the externality effect, where λ_1^g is the number of firms that use technology one in sector g. We further assume that $\Delta\alpha^r = \alpha_0^r - \alpha_1^r > 0$. We also denote $\Delta S^r = S_1^r - S_0^r$.

In general, we cannot fix the sign of ϕ from the outset. Both the negative and positive externalities remain as real-life options. If $\phi > 0$, then the green investment in sector g improves the relative profitability of the green technology in sector r. However, if $\phi < 0$, then the green investment in sector g congests the applications of green technology in sector r. In what follows, we derive our results under the positive technological externality, $\phi > 0$. We discuss the other option at the end of the study.

As in sector g, we take the number of firms in sector r as given. Let λ_0^{r+} denote the number of firms in sector r. Specifically, we assume that

$$B_0^r(\lambda_0^{r+}) = b_0^r + \theta_0^r - c_0^r \lambda_0^{r+} > 0.$$

The cut-off unit λ_1^{r+} is a firm that is indifferent between technologies zero and one in the absence of regulation. The unit is not fixed, but rather satisfies $B_0^r(\lambda_1^{r+}) = B_1^r(\lambda_1^{r+})$. As the profitability of technology one in sector r is influenced by the use of technology one in sector g, the size of λ_1^{r+} depends on the number of firms that use technology one in sector g. Then, the size of λ_1^{r+} depends on the fact whether sector g is regulated or not, so it depends on whether the number of units is λ_1^{g+} or λ_1^g in sector g. In what follows, we will assume that the policy treats the two sectors equally. That is, if it regulates sector g, then it regulates sector r as well. Then,

$$\lambda_1^{r+} = \frac{\Delta b^r - \Delta\theta^r + \phi\lambda_1^{g+}}{\Delta c^r}, \tag{3.11}$$

or, after utilizing Equation (3.4),

$$\lambda_1^{r+} = \frac{1}{\Delta c^r}\left(\Delta b^r + \frac{\phi}{\Delta c^g}\Delta b^g - \Delta\theta^r - \frac{\phi}{\Delta c^g}\Delta\theta^g\right). \tag{3.12}$$

After the regulation, the cut-off unit is denoted by λ_1^r, where

$$\lambda_1^r = \frac{\Delta b^r - \Delta\theta^r + s\Delta\alpha^r + \Delta S^r + \phi\lambda_1^g}{\Delta c^r}. \tag{3.13}$$

Furthermore, by Equations (3.5) and (3.12),

$$\lambda_1^r(\theta) = \lambda_1^{r+} + \frac{(\phi\Delta\alpha^g + \Delta c^g\Delta\alpha^r)}{\Delta c^g\Delta c^r}s + \frac{\Delta c^g\Delta S^r + \phi\Delta S^g}{\Delta c^g\Delta c^r}. \tag{3.14}$$

Finally, we make one more assumption: both technologies will always exist in both markets. This assumption mainly concerns the uncertainty outcomes. Effectively, we assume that uncertainty is small so that an entire technology will never exit. The assumption also concerns the size of the externality effect. We assume that the externality effect is small so that both technologies will exist in sector r.

3.2.4 Efficient Allocation of Emissions

We start our study of social choices by considering the choice under certainty. By the various sector- and technology-specific benefits from above, the aggregate benefits in the polluting sector amount to

$$B = \int_0^{\lambda_1^g} B_1^g d\lambda + \int_{\lambda_1^g}^{\lambda_0^{g+}} B_0^g d\lambda + \int_0^{\lambda_1^r} B_1^r d\lambda + \int_{\lambda_1^r}^{\lambda_0^{r+}} B_0^r d\lambda \tag{3.15}$$
$$= \int_0^{\lambda_1^g} (b_1^g - c_1^g \lambda) d\lambda + \int_{\lambda_1^g}^{\overline{\lambda}^g} (b_0^g - c_0^g \lambda) d\lambda$$
$$+ \int_0^{\lambda_1^r} (b_1^r - c_1^r \lambda + \phi \lambda_1^g) d\lambda + \int_{\lambda_1^r}^{\overline{\lambda}^r} (b_0^r - c_0^r \lambda) d\lambda.$$

In addition to the technological externality, another type of externality exists to which every firm in the industry contributes. It is a negative externality caused by the production. More specifically, we take it as homogenous pollution. Homogeneity means that only the aggregate amount of the externality matters, not the distribution of emitters.

Our interest lies in socially efficient allocation. In such an allocation, a policy will control both the technological externality and the pollution. In other words, the benefits in Equation (3.15) are maximal, given that the emissions are equal to

$$e = \int_0^{\lambda_1^g} \alpha_1^g d\lambda^g + \int_{\lambda_1^g}^{\lambda_0^{g+}} \alpha_0^g d\lambda^g + \int_0^{\lambda_1^r} \alpha_1^r d\lambda^r + \int_{\lambda_1^r}^{\lambda_0^{r+}} \alpha_0^r d\lambda^r \tag{3.16}$$
$$= \lambda_0^{g+} \alpha_0^g - \Delta\alpha^g \lambda_1^g + \lambda_0^{r+} \alpha_0^r - \Delta\alpha^r \lambda_1^r.$$

The efficient units, denoted by λ_e^g and λ_e^r, satisfy

$$\begin{cases} \Delta b^g - \Delta c^g \lambda_e^g + \phi \lambda_e^r + \mu \Delta\alpha^g = 0 \\ \Delta b^r - \Delta c^r \lambda_e^r + \phi \lambda_e^g + \mu \Delta\alpha^r = 0 \end{cases}, \tag{3.17}$$

or

$$\begin{bmatrix} -\Delta c^g & \phi \\ \phi & -\Delta c^r \end{bmatrix} \begin{bmatrix} \lambda_e^g \\ \lambda_e^r \end{bmatrix} = - \begin{bmatrix} \Delta b^g \\ \Delta b^r \end{bmatrix} - \mu \begin{bmatrix} \Delta\alpha^g \\ \Delta\alpha^r \end{bmatrix},$$

where μ is the shadow value of the emission constraint. The efficient units are

$$\begin{bmatrix} \lambda_e^g \\ \lambda_e^r \end{bmatrix} = \frac{1}{\Delta c^g \Delta c^r - \phi^2} \begin{bmatrix} -\Delta c^r & -\phi \\ -\phi & -\Delta c^g \end{bmatrix} \left(- \begin{bmatrix} \Delta b^g \\ \Delta b^r \end{bmatrix} - \mu \begin{bmatrix} \Delta \alpha^g \\ \Delta \alpha^r \end{bmatrix} \right),$$

or,

$$\lambda_e^g = \frac{\Delta c^r \Delta b^g + \phi \Delta b^r}{\Delta c^g \Delta c^r - \phi^2} + \mu \frac{\Delta \alpha_1^r \phi + \Delta \alpha^g \Delta c^r}{\Delta c^g \Delta c^r - \phi^2} \tag{3.18}$$

and

$$\lambda_e^r = \frac{\phi \Delta b^g + \Delta c^g \Delta b^r}{\Delta c^g \Delta c^r - \phi^2} + \mu \frac{\Delta \alpha^r \Delta c^g + \Delta \alpha^g \phi}{\Delta c^g \Delta c^r - \phi^2}. \tag{3.19}$$

We assume that $\lambda_e^g > 0$ and $\lambda_e^r > 0$, so $\Delta c^g \Delta c^r > \phi^2$.

Abatement cost function gives the maximal benefits for a given level of emissions. Consequently, this definition requires that the emission allocation is efficient. We know that, by definition,

$$\mu = \frac{dB}{de},$$

so $\mu(e)$ represents the marginal abatement function. We calculate in Appendix C.2 that

$$\mu(e) = cA - ce,$$

where A is a constant (independent of e) and

$$c = \frac{\Delta c^g \Delta c^r - \phi^2}{\Delta c^r (\Delta \alpha^g)^2 + 2\phi \Delta \alpha^g \Delta \alpha^r + \Delta c^g (\Delta \alpha^r)^2}. \tag{3.20}$$

The parameter c is the slope of the marginal abatement function. Especially note the way the positive spillover effect enters through ϕ into the slope parameter.

We already analyzed how the sectors will react to the regulation. Referring to this analysis (especially to Equations (3.5) and (3.13)), the cut-off units λ_1^g and λ_1^r satisfy

$$\begin{cases} \Delta b^g - \Delta c^g \lambda_1^g + s \Delta \alpha^g + \Delta S^g = 0 \\ \Delta b^r - \Delta c^r \lambda_1^r + \phi \lambda_1^g + s \Delta \alpha^r + \Delta S^r = 0 \end{cases}. \tag{3.21}$$

We show in Appendix C.3 that the efficient subsidy rule satisfies[11]

$$\Delta S^g = \frac{\Delta\alpha^g}{\Delta\alpha^r}\Delta S^r - \phi\lambda_1^r. \tag{3.22}$$

As a corollary, we show that

$$\mu = s + \frac{\Delta S^r}{\Delta\alpha^r}$$

at the efficient implementation. Thus, $\mu \neq s$ as long as $\Delta S^r \neq 0$.[12]

3.3 Social Welfare

3.3.1 Optimal Policy

An optimal policy maximizes expected social welfare. In Appendix C.4, we will solve the following problem:

$$\underset{\tau,\Delta S^g,\Delta S^r}{Max}\ EW = E\left[\int_0^{\lambda_1^g} B_1^g\, d\lambda^g + \int_{\lambda_1^g}^{\lambda_0^{g+}} B_0^g\, d\lambda^g + \int_0^{\lambda_1^r} B_1^r\, d\lambda^r + \int_{\lambda_1^r}^{\lambda_0^{r+}} B_0^r\, d\lambda^r\right]$$
$$-E\left[D(e(\lambda_1^g,\lambda_1^r)\right],$$

where $\lambda_1^g \equiv \lambda_1^g(\tau,\Delta S^g,\Delta S^r)$ and $\lambda_1^r \equiv \lambda_1^g(\tau,\Delta S^g,\Delta S^r)$ are the sector-specific responses calculated in Equations (3.5) and (3.14), respectively. The optimal policy consists of a tax rate (τ) and subsidies ($\Delta S^g, \Delta S^r$) for sectors g and r, respectively. We assume that the damages are homogenous, so they will depend on the aggregate level of emissions. In particular, we will assume that the damages are

$$D(e) = \frac{d}{2}e^2, \tag{3.23}$$

[11]Note that the efficiency supports subsidy profile

$$\Delta S^g = \frac{\Delta\alpha^g}{\Delta\alpha^r}\Delta S^r,$$

even if the positive externality remains absent ($\phi = 0$). We discuss this issue in depth in Chapter 1.

[12]We remind the reader about our discussion in Chapter 1 concerning the efficient subsidized implementations. In particular, the differences between efficient implementations were shown to be only nominal.

where $d > 0$, so we will concentrate on the quadratic damages of pollution that do not depend on uncertainty. We show that the optimal policy produces optimal expected cut-offs $E\lambda_1^g$ and $E\lambda_1^r$ that satisfy two conditions, namely

$$\tau - ED'(e) = \frac{-\Delta S^g + \phi E\lambda_1^r(\theta)}{\Delta\alpha^g}$$

and

$$\tau - ED'(e) = \frac{-\Delta S^r}{\Delta\alpha^r}.$$

Note that these conditions together imply

$$\Delta S^g = \frac{\Delta\alpha^g}{\Delta\alpha^r}\Delta S^r - \phi E\lambda_1^r(\theta).$$

Referring to Equation (3.22) above, this condition merely states that the emission allocation should be efficient.

Next, to keep the presentation fluent, we make the following assumption:

$$S_1^r = S_0^r = 0.$$

This assumption is a natural one, as it states that the policy should not substitute the spillover-receiving sector. Then, $\Delta S^r = 0$, and the optimal policy satisfies

$$\Delta S^g = \phi E\lambda_1^{r*}(\theta).$$

Thus, as for the regulation of the negative externality, it holds that

$$\tau = ED'(e). \tag{3.24}$$

3.3.2 Pegged Implementation of the Optimal Policy

Above, we solve the optimal policy as a combined tax-subsidy policy. Specifically, the implementation applies policy ΔS^g in the externality-generating sector, where $\Delta S^g \equiv S_1^g - S_0^g$. From here on, we will apply (another natural) assumption as we assume

$$\Delta S^g = S_1^g = S. \tag{3.25}$$

This assumption means that $S_0^g = 0$, so the old dirty technology is not subsidized at all. The assumption in Equation (3.25) does not restrict the generality of the analysis but it helps in reading it.

We say that the implementation

$$S = \phi E \lambda_1^r(\theta) \tag{3.26}$$

employs a lumpy payment. We consider next a rule that evolves as a function of the permit price. We write

$$E\lambda_1^r(\theta) = \frac{\phi \Delta b^g + \Delta c^g \Delta b^r}{\Delta c^g \Delta c^r - \phi^2} + \frac{\Delta \alpha^r \Delta c^g + \Delta \alpha^g \phi}{\Delta c^g \Delta c^r - \phi^2} ED'(e)$$

by using the optimality condition in Equation (C.13) in Appendix C.4. By Equation (3.24), it further holds that

$$\phi E\lambda_1^r(\theta) = \phi \frac{\phi \Delta b^g + \Delta c^g \Delta b^r}{\Delta c^g \Delta c^r - \phi^2} + \phi \frac{\Delta \alpha^r \Delta c^g + \Delta \alpha^g \phi}{\Delta c^g \Delta c^r - \phi^2} \tau. \tag{3.27}$$

This way of writing reveals the fundamental fact that the policy ultimately depends on the unit price of emissions. In particular, if we set $\Delta\theta^r = \Delta\theta^g = 0$ in rule $S = \phi\lambda_1^r(\theta)$, we have

$$S(s) = \phi \frac{\phi \Delta b^g + \Delta c^g \Delta b^r}{\Delta c^g \Delta c^r - \phi^2} + \phi \frac{\Delta \alpha^r \Delta c^g + \Delta \alpha^g \phi}{\Delta c^g \Delta c^r - \phi^2} s. \tag{3.28}$$

Consequently, we have payment rule $S(s)$ that is linear in s. It holds that

$$E[S(s)] = \phi \frac{\phi \Delta b^g + \Delta c^g \Delta b^r}{\Delta c^g \Delta c^r - \phi^2} + \phi \frac{\Delta \alpha^r \Delta c^g + \Delta \alpha^g \phi}{\Delta c^g \Delta c^r - \phi^2} Es,$$

so the rule satisfies the optimality condition in Equation (3.27) as long as $Es = \tau$. We will soon study the performance of rule $S(s)$ under uncertainty.

Note how the two externalities interact through variables ϕ and s in rule $S(s)$. If $s = 0$, then pollution is not an issue and the technological externality alone can be internalized by the constant part of rule $S(s)$. On the other hand, if the technological externality does not exist, then $\phi = 0$, so $S = 0$. For future references, we denote

$$S(s) = \Gamma + \Gamma_s s, \tag{3.29}$$

where $\Gamma > 0$ and $\Gamma_s > 0$.

We also like to comment on the efficiency of the implementation *ex-post*, that is, when uncertainty has turned into a set of publicly known parameters. According to our earlier calculations, efficiency means that a condition $S = \phi\lambda_1^r(\theta)$ holds. However, by definition,

$$\lambda_1^r(\theta) = \frac{S(s)}{\phi} - \frac{(\Delta c^g \Delta\theta^r + \phi\Delta\theta^g)}{\Delta c^g \Delta c^r - \phi^2}.$$

As we rule out contingent payment rules (i.e., rules contingent on the outcomes of uncertainty), the allocation inevitably becomes inefficient *ex-post*.[13]

3.4 Prices Versus Quantities

3.4.1 The Framework

We start our investigation by providing a framework that we will apply throughout the rest of the study. The framework consists of (uncertain) benefits and damages as functions of the emissions price and the subsidies.[14] We assumed above a specific efficient implementation that provides a positive subsidy S for the externality generators and a zero subsidy for the others. We incorporate this fact into the model by rewriting Equation (3.5) as

$$\begin{aligned} \lambda_1^g &= \frac{\Delta b^g - \Delta\theta^g + s\Delta\alpha^g + S}{\Delta c^g} = \lambda_1^{g+}(\theta) + \frac{\Delta\alpha^g}{\Delta c^g}s + \frac{S}{\Delta c^g} \\ &= \lambda_1^{g+}(\theta) + z_g s + \gamma_g S \end{aligned} \tag{3.30}$$

for subsidy S. Furthermore, the cut-off unit in sector r becomes (Equation (3.14))

[13]We remark that this state is not normally observed in the studies of instrument choice. Rather, the efficiency condition is strictly deterministic. It should be evident that the spillover effect among the polluting industry is responsible for the new state.

[14]The policy cannot internalize the technological externality by firm-specific technology rules. In essence, the regulatory agency lacks firm-specific information. The industry should use both brown and green technologies, but the agency cannot distribute the technology standards in the absence of firm-specific data. Naturally, a uniform technology standard does not work either.

$$\lambda_1^r = \frac{\Delta b^r - \Delta\theta^r + s\Delta\alpha^r + \phi\lambda_1^{g*}(\theta)}{\Delta c^r} \tag{3.31}$$
$$= \lambda_1^{r+}(\theta) + \frac{\Delta\alpha^r + \phi z_g}{\Delta c^r} s + \frac{\phi\gamma_g}{\Delta c^r} S = \lambda_1^{r+}(\theta) + z_r s + \gamma_r S.$$

Especially note how subsidies flow indirectly into sector r through the technological externality. Even though the policy does not subsidize units in sector r, the subsidization in sector g benefits these units.

We use $B(s,S,\theta)$ to denote uncertain benefits. We incorporate the cut-off units from Equations (3.30) and (3.31) into benefits in Equation (3.15). At the most general level, we write

$$B(s,S,\theta) = \Psi^1(s,S) + \Psi^2(s,S,\theta), \tag{3.32}$$

where Ψ^1 is totally independent of the uncertainties while Ψ^2 is not. We write the deterministic part in Appendix C.5 (see Part I, Equation (C.17)) as

$$\Psi^1(s) = z(S) + z_1 s + z_2(S)s - \frac{z_3}{2}s^2, \tag{3.33}$$

where factors $z(S)$ and $z_2(S)$ depend on subsidy S. We also write the stochastic part as

$$\Psi^2(s,\theta) = y(\theta) - y_1(\theta)(S + \Delta\alpha^g s) \tag{3.34}$$

in Appendix C.5 (see Part III, Equation (C.26)). In particular, we show in there that $E[y(\theta)] \neq 0$ and $E[y_1(\theta)] = 0$.

We wrote the emissions formula above in Equation (3.16). We incorporate the cut-off units from Equations (3.30) and (3.31) into the formula. The emissions in terms of s and S are

$$e(s,S,\theta) = x + \left(\frac{\Delta\alpha^g\Delta c^r + \Delta\alpha^r\phi}{\Delta c^r\Delta c^g}\right)\Delta\theta^g + \frac{\Delta\alpha^r}{\Delta c^r}\Delta\theta^r \tag{3.35}$$
$$-\left(\frac{\Delta\alpha^g\Delta c^r + \Delta\alpha^r\phi}{\Delta c^r\Delta c^g}\right)S - \left(z_r\Delta\alpha^r + \Delta\alpha^g z_g\right)s$$
$$= x + x(\theta) - x_1 S - x_2 s.$$

Specifically, the term x does not include either uncertainties, subsidies S, or the price

s (see Appendix C.5, Equation (C.27)), and

$$x(\theta) = \left(\frac{\Delta\alpha^g \Delta c^r + \Delta\alpha^r \phi}{\Delta c^r \Delta c^g} \right) \Delta\theta^g + \frac{\Delta\alpha^r}{\Delta c^r} \Delta\theta^r. \tag{3.36}$$

Clearly, $E[x(\theta)] = 0$.

3.4.2 The Pegged Implementation

Consider the optimal pegged implementation. We will consider first the implementation by tax policy ($s = \tau$) and after that by tradable permits ($s = p$). We start by inserting "the pegged rule" $S(s)$ from Equation (3.29) into the representations of Equations (3.32) and (3.35). Now, it holds that

Lemma 2

$$\Psi^1(s) = \overline{\Psi}^1 - \frac{1}{2c} s^2 \tag{3.37}$$

and the level of emissions equals

$$e(s;\theta) = X + x(\theta) - \frac{s}{c}, \tag{3.38}$$

where $\overline{\Psi}^1$ *and* X *are constants (do not depend on* s *or* θ*),* $x(\theta)$ *is given by Equation (3.36), and* c *is given by Equation (3.20).*
Proof. *See Appendix C.5 for Parts II and IV and Equations (C.24) and (C.28).* ■

We will show next that the pegged implementation truly reproduces the optimal rule (as presented in Equation (3.24)). The optimal policy satisfies (by Equations (3.33), (3.34), and (3.35))

$$\frac{dEW}{d\tau} = \frac{dE[B-D]}{d\tau} = E\left[\frac{d\Psi^1(\tau)}{d\tau} + \frac{\Psi^2(\tau,\theta)}{d\tau} - de\frac{de(\tau,\theta)}{d\tau} \right] = 0. \tag{3.39}$$

By Equation (3.34),

$$E\frac{\Psi^2(\tau,\theta)}{d\tau} = E\left[y_1(\theta) \left(\frac{dS(\tau)}{d\tau} + \Delta\alpha^g \right) \right].$$

As subsidy $S(\tau)$ is as constant, we have

$$E\frac{\Psi^2(\tau,\theta)}{d\tau}=\left(\frac{dS(\tau)}{d\tau}+\Delta\alpha^g\right)E\left[y_1(\theta)\right],$$

so $E\frac{\Psi^2(\tau,\theta)}{d\tau}=0$. By this information and by Lemma 2, we write the condition in Equation (3.39) as

$$E\left[\frac{\tau}{c}-\frac{de}{c}\right]=0,$$

so

$$\tau=d\overline{e}.$$

Thus, the expected price should be equal to the expected marginal damage under the pegged design.[15]

If the policy applies tradable permits, the permit market equilibrium satisfies $l=e$, so (by Equation (3.35))

$$l=x+x(\theta)-x_1S(p)-x_2p, \tag{3.40}$$

where l is the total number of permits in the market and p is the permit price. In particular, as $S(p)$ is a linear rule (Equation (3.29)), it holds that

$$p=\overline{p}+\frac{x(\theta)}{x_2}.$$

As a deterministic relationship exists between $\overline{p}$ and l, we can take $\overline{p}$ as the policy variable as well. Then,

$$\frac{dEW}{d\overline{p}}=E\left[\frac{d\Psi^1(p)}{d\overline{p}}+\frac{\Psi^2(p,\theta)}{d\overline{p}}-de\frac{de(p,\theta)}{d\overline{p}}\right]=0.$$

Specifically,

$$E\frac{\Psi^2(p,\theta)}{d\overline{p}}=E\left[y_1(\theta)\left(\frac{\overline{p}+x(\theta)}{d\overline{p}}+\Delta\alpha^g\right)\right]=(1+\Delta\alpha^g)E\left[y_1(\theta)\right]=0.$$

Thus,

$$\frac{dEW}{d\overline{p}}=E\left[\frac{\overline{p}}{c}-\frac{de}{c}\right]=0,$$

[15]The bar above represents the expected value.

so the first-best permit policy satisfies

$$\overline{p} = d\overline{e}.$$

3.4.3 Second-Best Design

We are also interested in a second-best setting, where the agency identifies the technological externality but it cannot directly control it. As direct internalization is not an option, the agency will only indirectly control it. This is possible, as the agency has the mandate to control the negative externality, that is, the pollution.

We incorporate the second-best setting into the model by setting $S = 0$. We insert the value $S = 0$ into Equations (3.33), (3.34), and (3.35). It follows that the non-stochastic part of the benefits is

$$\Psi^1_{sb}(s) = z(0) + z_1 s - \frac{z_3}{2} s^2,$$

while

$$e(s,\theta) = x + x(\theta) - x_2 s. \tag{3.41}$$

The second-best design then satisfies

$$\frac{dEW}{d\tau} = \frac{dE[B-D]}{d\tau} = E\left[z_1 - z_3 s - \frac{de(\tau,\theta)}{d\tau} \right] = 0$$

or

$$\tau_{sb} = \frac{x_2}{z_3} d\overline{e} + \frac{z_1}{z_3}.$$

Clearly, τ_{sb} deviates from the first-best rule. Note also that we can derive the second-best permit policy $\overline{p}_{sb} = \tau_{sb}$ along the lines of an earlier section.

Before closing the section, we quickly review the way the marginal benefits behave in the first- and second-best designs. In the first-best design, we denote

$$c = 1 / \frac{d^2 \Psi^1(s)}{ds^2}.$$

We already interpreted the constant (see Equation (3.20) above) as the slope of the augmented abatement cost function. We employ the term “augmented" as the slope parameter records the influence of the technological externality. Actually, quoting

Equation (3.20), the influence of the technological externality lies in parameter $\phi \neq 0$. If $\phi = 0$, then c becomes the slope of the standard abatement cost function. It is a straightforward task to show that the slope of the abatement function gets less steep when a positive technological externality steps in.

In the second-best design,

$$\frac{d\Psi^1_{sb}(s)}{ds} = z_1 - z_3 s.$$

Define

$$c_{sb} = \frac{1}{z_3}, \tag{3.42}$$

where (by Equation (C.18) in Appendix C.5)

$$z_3 = \Delta c^r (z_r)^2 - 2\phi z_g z_r + \Delta c^g \left(z_g\right)^2.$$

After (some laborious) expanding (z_g and z_r are defined in Equations (3.30) and (3.31), respectively), we write

$$c_{sb} = \frac{\Delta c^g \Delta c^r}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 - \frac{\phi^2}{\Delta c^g} (\Delta \alpha^g)^2}. \tag{3.43}$$

Clearly, $c_{sb} > c$. If $\phi = 0$, then c_{sb} becomes again the slope of the standard abatement cost function.

3.4.4 Pegged Comparative Advantage

It is time to move to the main subject of our study, namely, to the instrument choice under uncertainty. An integral part of the model is the so-called Weitzman assumption. According to this assumption, the agency sets the policy parameters first. Then, the regulated firms react to the uncertainty and the game ends. That is, both parties move only once. Another question concerns the set of feasible instruments. We have already assumed that the agency may choose between an environmental tax and tradable permits. A further question concerns the presence of the technological externality and the subsidy rule. We will apply the so-called pegged payment, which is a function of the emissions price. Consequently, if the price of the emissions is fixed, then payment S becomes fixed as well. We will discuss alternative implementations later on.

We will base the instrument choice on the comparative advantage (Weitzman [84]). We denote it by Δ. In choosing between instruments τ and p, we have

$$\begin{aligned}\Delta(I,J) &= EB(I(\theta),\theta) - ED(e(I(\theta),\theta)) - [EB(J(\theta),\theta) - ED(e(J(\theta),\theta))] \\ &= EB(I(\theta),\theta) - EB(J(\theta),\theta) - [ED(e(I(\theta),\theta)) - ED(e(J(\theta),\theta))].\end{aligned} \tag{3.44}$$

Specifically, a strictly positive $\Delta(I,J)$ implies that the agency prefers the instrument $I(\theta)$.

We let $I(\theta) = \tau$ and $J(\theta) = p(\theta)$. We calculate next the differences in benefits and damages in the comparative advantage. The subsidy follows the rule in Equation (3.29). As we denote the permit price in this design by p, then the subsidy is

$$S(p) = \Gamma + \Gamma_s p. \tag{3.45}$$

The permit price is determined by the market, where the demand of permits equals the supply. The equilibrium (see Equation (3.40)) satisfies

$$l = x + x(\theta) - x_1 S(p) - x_2 p,$$

or, by Lemma 2,

$$l = X + x(\theta) - \frac{1}{c} p.$$

The equilibrium price then satisfies

$$p(\theta) = \tau + cx(\theta). \tag{3.46}$$

Recall that $Ep(\theta) = \tau$, where τ is the tax rate.

Next, the representation in Equation (3.32) states that

$$B(s,\theta) = \Psi^1(s) + \Psi^2(s,\theta).$$

Inserting factor $\Psi^2(s,\theta)$ (Equation (3.34)) together with rule $S(p)$ (Equation (3.45)) into the benefits gives us

$$B(s,\theta) = \Psi^1(s) + y(\theta) - y_1(\theta)(\Gamma + \Gamma_s p + \Delta\alpha^g p),$$

where $\Psi^1(s)$ is given by Equation (3.37) in Lemma 2. We further denote

$$\eta = (\Gamma_s + \Delta\alpha^g) > 0 \tag{3.47}$$

and

$$\Xi(\theta) = y_1(\theta)\Gamma, \tag{3.48}$$

so

$$B(s,\theta) = \Psi^1(s) + y(\theta) - \Xi(\theta) - y_1(\theta)\eta s. \tag{3.49}$$

We pay specific attention to the cross product between uncertainty and unit price, namely to the product $y_1(\theta)s$. This product will mainly explain the forthcoming new effect in instrument choice. As for the expected benefits under tradable permits, the permit price process follows Equation (3.46) and $\Psi^1(s)$ follows Equation (3.37). We then have

$$EB(p(\theta),\theta) = \overline{\Psi}^1 - \frac{c}{2}E\left(x(\theta)\right)^2 + E\left[y(\theta) - \Xi(\theta) - y_1(\theta)\eta\tau\right] - c\eta E y_1(\theta)x(\theta). \tag{3.50}$$

The tax design fixes the price of the emissions so that $s = \tau$. The subsidy rule becomes

$$S(\tau) = \Gamma + \Gamma_s \tau,$$

so the subsidy is a fixed entity. The expected benefits in the tax regime are

$$EB(\tau) = \overline{\Psi}^1 + E\left[y(\theta) - \Xi(\theta) - y_1(\theta)\eta\tau\right],$$

so the difference becomes

$$EB(\tau,\theta) - EB(p(\theta),\theta) = c\eta E y_1(\theta)x(\theta) + \frac{c}{2}E\left(x(\theta)\right)^2. \tag{3.51}$$

Emissions remain fixed in the quantity instrument implementation. With taxes, we apply Equation (3.35), so that

$$e(\tau,\theta) = x + x(\theta) - \left(\frac{\Delta\alpha^g \Delta c^r + \Delta\alpha^r \phi}{\Delta c^r \Delta c^g}\right) S(\tau) - \left(z_r \Delta\alpha^r + \Delta\alpha^g z_g\right)\tau,$$

or, by Lemma 2,

$$e(\tau,\theta) = X + x(\theta) - \frac{\tau}{c} = \bar{e} + x(\theta). \tag{3.52}$$

We then calculate the difference in expected damages as

$$E\left[D(e(\tau,\theta)) - D(e(p(\theta))\right] = \frac{d}{2} E\left(x(\theta)\right)^2. \tag{3.53}$$

We incorporate the differences in Equations (3.51) and (3.53) into the comparative advantage in Equation (3.44). We have

Proposition 5 *Let $c > 0$ be the slope of the marginal abatement function, $d > 0$ be the slope of the marginal damage function, and $\phi > 0$ be the externality effect. Furthermore, assume that the efficient subsidy rule in Equation (3.45) internalizes the technological externality. Then, the comparative advantage between the price and the tax instrument is*

$$\Delta(\tau, p) = \frac{c-d}{2} E\left(x(\theta)\right)^2 + c\eta E y_1(\theta) x(\theta), \tag{3.54}$$

where $c\eta E y_1(\theta) x(\theta) > 0$.

The measure $\Delta(\tau, p)$ consists of two additive terms. The term $\frac{c-d}{2} E\left(x(\theta)\right)^2$ is the traditional Weitzman effect. Accordingly, the instrument choice depends on the slopes of the abatement cost function (c) and the damage function (d). The second term is $c\eta E y_1(\theta) x(\theta)$. By Equation (C.25) in Appendix C.5,

$$y_1(\theta) = \frac{\phi}{\Delta c^g \Delta c^r}\left(\Delta\theta^r + \frac{\phi}{\Delta c^g}\Delta\theta^g\right),$$

so, if $\phi = 0$, then $y_1(\theta) = 0$. Thus, if no technological externality exists, then the second term in the measure $\Delta(\tau, p)$ does not exist.

Overall, if the knowledge spillover disappears, the comparative advantage reduces to a standard Weitzman [84] measure. First, as we just noted, the value $\phi = 0$ implies that $y_1(\theta) = 0$. Second, in the absence of the spillover effect, the slope of the abatement function reduces to a standard slope denoted by c_z (see Equation (3.58) below). Third, the variance $E\left(x(\theta)\right)^2$ reduces to $E\left(x_z(\theta)\right)^2$, which by Equation (3.36) equals

$$E\left(x_z(\theta)\right)^2 = E\left(\frac{\Delta\alpha^g}{\Delta c^g}\Delta\theta^g + \frac{\Delta\alpha^r}{\Delta c^r}\Delta\theta^r\right)^2.$$

In summary, the comparative advantage reduces to a standard Weitzman [84] measure,

$$\Delta(\tau, p) = \frac{c_z - d}{2} E\left(x_z(\theta)\right)^2.$$

3.4.5 Graphical Illustration

The result just derived is not standard, so further illustration is needed. We do this graphically, and, for the sake of an easy exposition, we draw the figure in terms of the quantity (the emissions). The exact shape of the marginal benefit curve is not obvious at the outset. We base the figure on the previous section and, specifically, on the application of Lemma 2.

Recall that we operate in a pegged regime. The relation between price s and the quantity e can be written (by Equation (3.38)) as

$$X + x(\theta) - \frac{s}{c} - e = 0. \tag{3.55}$$

We solve $s(e;\theta)$ and insert it into the benefits in Equation (3.32). Specifically, by Equation (3.37),

$$\Psi^1(s(e;\theta)) = \overline{\Psi}^1 - \frac{1}{2c}\left(s(e;\theta)\right)^2,$$

and by Equation (3.49),

$$\Psi^2(s(e;\theta), \theta) = y(\theta) - \Xi(\theta) - y_1(\theta)\eta s(e;\theta),$$

so,

$$\begin{aligned} mb(e;\theta) &= \frac{dB(s(e);\theta)}{de} = \left(\frac{d\Psi^1(s(e;\theta))}{de} + \frac{\Psi^2(s(e;\theta), \theta)}{de}\right)\frac{ds(e;\theta)}{de} \\ &= c\eta y_1(\theta) + s(e;\theta), \end{aligned}$$

or

$$mb(e;\theta) = cX + cx(\theta) + c\eta y_1(\theta) - ce. \tag{3.56}$$

We see the in special pattern how uncertainty (θ) shifts the marginal benefit function.

To understand the previous comparative advantage (Equation (3.54)), one has to understand a key feature of the standard model first. By standard model, we refer to a model without technological externality, which is to say that $\phi = 0$. The key feature is that the marginal benefit function $mb(e;\theta)$ and the price function $s(e;\theta)$ are one and the same thing.[16] Conversely, in our model,

$$mb(e;\theta) - s(e;\theta) = c\eta y_1(\theta) \neq 0. \tag{3.57}$$

As long as $\phi = 0$, then $y_1(\theta) = 0$ and $mb(e;\theta) = s(e;\theta)$. Note further that the emissions in the tax regime are determined by the price function. Equation (3.55) then gives us the emissions response

$$e(\tau;\theta) = cX + x(\theta) - \frac{\tau}{c}.$$

In Figure 3.2, the tax policy is a horizontal line at τ, while the permit policy is represented by a vertical line placed at $e = l$. The uncertainty that we assume is plain, as there are only two possible states of the world. Please note that the subscript d stands for low realization of uncertainty, while the subscript u represents high realization of uncertainty. We denote the marginal damage curve by md. By assumption, it remains stable.

Figure 3.2 represents the discrepancy between the marginal benefits (the mb-curves) and the price functions (the s-curves). Referring back to Equation (3.56), the factor $c\eta y_1(\theta)$ shifts the mb-curve away from the s-curve.[17] The figure also shows that the market tax response is either e_d or e_u. Had they been determined by the marginal benefit responses (as they would in a standard model), they would have been e_{mbd} and e_{mbu}, respectively. The quota is fixed, so the emissions under the quantity system stay fixed at $e = l$.

[16] We already reviewed this feature in a different context (see Section 1.4.5, Chapter 1).

[17] The expected curves would lie on top of each other. This follows, as $Ey_1(\theta) = 0$. Furthermore, they would cross the point where the vertical line l and the horizontal line τ cross.

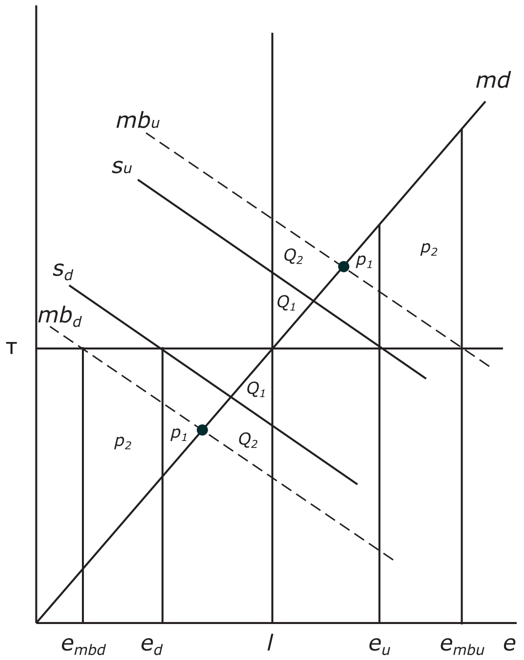

Figure 3.2 Prices vs. Quantities under a Positive Technological Externality That Is Internalized in an Efficient Way

We have also incorporated the *ex-post* optimal policies into Figure 3.2. If the agency could reset the policy, it would follow the black dots in the figure. The commitment to a certain instrument creates a certain welfare loss. In Figure 3.2, the quantity policy loss is equal to an area $Q_1 + Q_2$, while the loss with tax policy is equal to p_1. In particular, note the influence of the technological externality. In the absence of it, there would have been an additional area equal to p_2 that would have been counted as a cost to the price instrument. In this respect, the pegged policy favors the price instrument.

Mendelsohn [42] studies endogenous technological change and instrument choice under uncertainty. In his model, innovations in clean technology make the benefit curve less steep, that is, less responsive to the changes in price. Mendelsohn compares comparative advantages with and without R&D. If we want to interpret our results in a similar manner, we should compare regimes with and without positive spillovers. In practice, one has to control the differences between two separate slopes of the abatement functions. In this approach, we write

$$c_z = \frac{\Delta c^g \Delta c^r}{\Delta c^r (\Delta \alpha^g)^2 + \Delta c^g (\Delta \alpha^r)^2} > 0 \tag{3.58}$$

as the slope of the abatement function in the absence of spillovers. Using this, we write a new version of the comparative advantage as

$$\Delta(\tau, p) = \frac{1}{n}\left(\frac{c_z - nd}{2} E\,(x(\theta))^2 + c_z \eta E y_1(\theta) x(\theta)\right), \tag{3.59}$$

where

$$n = \frac{c_z}{c} = \frac{\Delta c^g \Delta c^r}{\Delta c^g \Delta c^r - \phi^2} \frac{\Delta c^r (\Delta \alpha^g)^2 + \Delta c^g (\Delta \alpha^r)^2 + 2\phi \Delta \alpha^g \Delta \alpha^r}{\Delta c^r (\Delta \alpha^g)^2 + \Delta c^g (\Delta \alpha^r)^2}.$$

As both terms in the multiplication are strictly greater than zero, then $n > 1$. In Equation (3.59), the term outside the brackets is strictly larger than zero, so it does not affect the sign of the measure. On the contrary, the coefficient n inside the brackets favors the quantity instrument.

Factor n catches the same kind of effect that Mendelsohn discusses. That is, it catches the effect that a new impact (investment, spillovers) makes the slope of the abatement function less steep.[18] We denote this effect as a slope effect. It belongs to a larger group of effects that we labeled as the base effects in the Introduction. These effects follow as we move between representations that apply efficient implementations. In Chapter 2, we have another base effect (also denoted by n), namely, the so-called scope effect. It is calculated between perfect or imperfect participations in regulation.

In Mendelsohn [42], the benefits of investments are purely private, so subsidization is not required for efficiency reasons. Consequently, effects like cost effects do not evolve there. In Figure 3.2, instead, the price function shifts away from the marginal benefit function because of the cost effect. We already discussed in Chapter 1 that the price function and the marginal benefit function are two separate entities under inefficient subsidization. We showed that the price function tilts away from the marginal benefit function at the horizontal axis (see Figure 1.2). However, there is a fundamental difference between the chapters. In Chapter 1, expected benefits and price functions are genuinely separate entities, while here, these functions di-

[18]Note that some writers (see Requate [61]) interpret the presence of spillovers as a form of R&D.

verge only *ex-post.* We discussed in the Introduction how efficiency is ruined *ex-ante* in Chapter 1, while here, efficiency is developed *ex-ante* but ruined *ex-post*.

3.4.6 Second-Best Comparative Advantage

We defined the second-best design above as a policy in which the technological externality is controlled only indirectly. In particular, subsidies are not an option in the design. This means that the second-best policy is studied by setting $S = 0$ in our framework. In this section, we are interested in the instrument choice in the second-best setting. We will calculate the value of comparative advantage as presented in Equation (3.44). Here, it corresponds to

$$\Delta(\tau_{sb}, p_{sb}) = EB(\tau_{sb}, \theta) - EB(p_{sb}(\theta), \theta) - [ED(e(\tau_{sb}, \theta)) - ED(l_{sb})], \quad (3.60)$$

where p_{sb} is the permit price, τ_{sb} is the tax rate, and l_{sb} is the quota in the second-best design. We discussed the determination of τ_{sb} and l_{sb} in Section 3.4.3.

In the permit markets, the market equilibrium will satisfy (by Equation (3.41))

$$l_{sb} = x + x(\theta) - x_2 s,$$

so the equilibrium permit price is

$$p_{sb}(\theta) = \frac{1}{x_2}(x + x(\theta) - l_{sb}).$$

As $E[p_{sb}(\theta)] = \tau_{sb}$, we may also write

$$p_{sb}(\theta) = \tau_{sb} + \frac{x(\theta)}{x_2} = \tau_{sb} + w x(\theta).$$

Furthermore, by Equation (3.35),

$$w = \frac{1}{x_2} = \frac{1}{z_r \Delta\alpha^r + z_g \Delta\alpha^g}, \quad (3.61)$$

or, by expanding, (we define z_g and z_r in Equations (3.30) and (3.31), respectively),

$$w = \frac{\Delta c^g \Delta c^r}{\Delta c^g (\Delta\alpha^r)^2 + \phi \Delta\alpha^r \Delta\alpha^g + \Delta c^r (\Delta\alpha^g)^2}. \quad (3.62)$$

By comparing Equations (3.43) and (3.62), we see that $w \neq c_{sb}$.

After incorporating the value $S = 0$ into representations in Equations (3.33) and (3.34), the benefits in the second-best design are

$$B(s,\theta) = \Psi^1(s) + \Psi(s,\theta) = z(0) + z_1 s - \frac{1}{2c_{sb}} s^2 + y(\theta) - y_1(\theta)\Delta\alpha^g s.$$

We further incorporate the permit price ($s = p_{sb}(\theta)$) into the benefits and calculate the expected benefits as

$$\begin{aligned} &EB(p_{sb}(\theta),\theta) \\ &= \overline{\Psi}^1 + E\left[y(\theta) - y_1(\theta)\tau_{sb}\right] - w\Delta\alpha^g E y_1(\theta)x(\theta) - \frac{w^2}{2c_{sb}} E\left(x(\theta)\right)^2. \end{aligned}$$

Specifically,

$$\begin{aligned} EB(p_{sb}(\theta),\theta) - EB(\tau_{sb},\theta) &= w\Delta\alpha^g E y_1(\theta)x(\theta) - \frac{w^2}{2c_{sb}} E\left(x(\theta)\right)^2 \\ &= \frac{w}{c_{sb}}\left(c_{sb}\Delta\alpha^g E y_1(\theta)x(\theta) - \frac{w}{2} E\left(x(\theta)\right)^2\right). \end{aligned} \tag{3.63}$$

On the other hand, according to Equation (3.41), the emissions in the tax system are

$$e(\tau_{sb},\theta) = \overline{e}_{sb} + x(\theta),$$

while the emissions with tradable permits are fixed and equal to $\overline{e}_{sb}$. Together, these give us the difference in expected damages as

$$E\left[D(e(\tau_{sb},\theta)) - D(l_{sb})\right] = \frac{d}{2} E\left(x(\theta)\right)^2. \tag{3.64}$$

Notably, the difference in expected damages is precisely the same as in the first-best comparative measure above (see Equation (3.53)). Altogether, after incorporating the differences in benefits (Equation (3.63)) and in damages (Equation (3.64)) into the comparative advantage (Equation (3.60)), we have

$$\Delta(\tau_{sb}, p_{sb}) = \left(\frac{w^2}{c_{sb}} - d\right)\frac{E\left(x(\theta)\right)^2}{2} + w\Delta\alpha^g E y_1(\theta)x(\theta). \tag{3.65}$$

The second-best comparative measure has a similar structure to its first-best counterpart in Equation (3.54). In addition to the usual slope comparison, an additive (positive) cross product emerges. Thus, even though the technological uncertainty remains unpriced, the additive cross product remains in the comparative advantage. Another question concerns the qualitative nature of this new measure. One may wonder how does the second-best design affects the comparative advantage as compared to original measure by Weitzman [84]. To answer this, we first write the comparative advantage as

$$\Delta(\tau_{sb}, p_{sb}) = \frac{r_1 c - d}{2} E\,(x(\theta))^2 + c\, r_2 E y_1(\theta) x(\theta),$$

where

$$r_1 = \frac{w^2}{c c_{sb}}$$

and

$$r_2 = \frac{w \Delta \alpha^g}{c}.$$

We have

Lemma 3 *i)* $r_1 > 1$ *and ii)* $r_2 > 0$.

Proof. The non-negativity of r_2 follows from the non-negativity of w, $\Delta\alpha^g$, and c. As for the size of factor r_1, see Appendix C.6. ■

Thus, as compared to the mere standard outcome, where $r_1 = 1$ and $r_2 = 0$, the implemented policy will favor the price instrument.

3.5 Concluding Remarks

We extend the workings of traditional market-based instruments (i.e., tradable permits and environmental taxes) to cover subsidization as well. This is required as we assume that positive externality (knowledge spillovers) exists within the polluting industry. We show how pricing only the negative externality (pollution) and setting the price of positive externality to zero does not produce an optimal societal outcome. We further follow the framework by Weitzman [84], which is essentially a second-best study of regulation under uncertainty. It asks, whether the environmental agency should commit to a price instrument or a quantity instrument, when the agency is forced to make a binding commitment in its policy. The agency cannot reset the policy parameters once new information arrives or it cannot base the regulation on state-contingent contracts. We show that these means are especially required here to secure the efficiency of emission allocation. As subsidization inevitably becomes inefficient, it affects the choice between prices and quantities. In particular, the inefficiency is reflected in the cost effect of the instrument choice, and like in the earlier chapters, the cost effect will favor the price instrument, that is environmental taxation.

The implementation in this chapter uses payments that consist of two parts: the environmental payment and the subsidy payment. A specialty in the optimal policy analysis is the fact that it clearly promotes subsidization but does not explicitly state the form of subsidization. This fact carries a close resemblance to the original analysis of Weitzman, where the optimal policy promotes the pricing of emissions but does not explicitly state whether prices or quantities should be applied. We end up promoting a linear rule in subsidization that explicitly depends on the unit price of emissions. In particular, when this rule is incorporated into the system of tradable permits, the subsidy will depend on the permit price. In comparing prices and quantities, we compare implementations where one implementation (quantities) applies a stochastic subsidy rule while the other (prices) applies a fixed subsidy rule.

We call the stochastic subsidy rule the pegged subsidy rule. The name originates from the currency markets, where a currency peg represents one kind of exchange rate policy. Under the policy, typically one (small) currency follows another (big) one in a fixed relationship. However, as we explained above, the optimal policy does not fix the type of the subsidy implementation. This gives us reason to look after rules outside the pegged subsidy rule. First, we may consider a kind of hybrid system in which the price of emissions is determined in the markets, but the subsidy is fixed and equal to the subsidy under taxes. In other words, the permit policy would apply the rule (see Equation (3.29))

$$S(\tau) = \Gamma + \Gamma_s \tau, \tag{3.66}$$

where τ is a tax rate. Alternatively, as we do in Chapters 1 and 2, we may analyze a rule that utilizes emission thresholds in subsidization. In the present context, a unit is entitled to an emission threshold l_i^k as long as it applies technology i in sector k . Consequently, the unit is entitled to a subsidy payment equal to

$$S(p) = p l_i^k.$$

In particular, in the policy above, the threshold-based subsidy would be

$$S(p) = p l_1^g \tag{3.67}$$

as the subsidy is paid only to a polluting unit that uses green technology in the externality-generating sector.

We do not analyze these alternatives at length but rather leave the issue for future reviews. However, according to our preliminary calculations, the functioning of the hybrid system (subsidy rule in Equation (3.66)) is not standard. In particular, we have calculated that the comparative advantage resembles the second-best comparative advantage that we derive in Equation (3.65). As for the other implementation, namely, the threshold case (depicted in Equation (3.67)), we remind the reader about our extensive discussion about the threshold implementations in Chapter 1. Based on that discussion, we note how threshold implementations may create a volume effect. The volume effect is absent in the analysis of this chapter only because the permit endowment is entirely auctioned off. Even though the number of firms fluctuates between different technologies in different sectors, the level of emissions stays constant throughout the entire study.

Finally, we remark about the possibility that a negative externality could prevail among the polluting industry. We have briefly analyzed this possibility. As far as optimality requires (see Equation (3.26)) that $S = \phi E \lambda_1^r(\theta)$, we have $\phi < 0$ and $E \lambda_1^r(\theta) > 0$, so $S < 0$. This means that the optimal policy should tax firms in sector g because of their externality generation. Furthermore, the value $\phi < 0$ can be shown to affect one of the main results of this chapter, namely, Proposition 5. In particular, the negative technological externality gives rise to a series of complex impacts. This is in contrast to the case of positive technological externality, which yields much more predictable outcomes.

Overall, our interest lies in regulation in a situation where both negative and positive externalities exist simultaneously. We consider knowledge spillover as a positive externality. The properties of knowledge spillover are fairly intuitive and uncontroversial, at least in theory. The accurate identification and measurement of the effect is a bigger issue. In this respect, our approach does not question the identification of the effect but rather takes the spillover effect as a known parameter. A natural extension would be to incorporate an uncertain spillover effect into the problem of instrument choice. Instead of concentrating on uncertain marginal benefits and damages, we might focus on the uncertain spillover effect. Our current framework would need considerable elaboration in order to conform to such an approach.

CONCLUSIONS

In the opening part of his 1965 book "*Theory of Production*", Ragnar Frisch [19], the co-recipient of the first Nobel Memorial Prize in Economic Sciences, reviews various basic concepts of production. Among other things, he discusses the case of joint production: "*In this case the technical process itself contains an element which makes it impossible (or very difficult) to produce one product without at the same time producing one or more other products,*" (Frisch [19], p. 11). He draws a further distinction between main products, bi-products, and waste products in joint production. He ends up speculating that "*a change in the price situation may result in the bi-product or even waste product being elevated to the status of main product,*" (Frisch [19], p. 11).

We think that the general attitude toward the environment has changed. Nowadays, the quality of air, land and water is a big issue. We do not see the burning of coal as a single production of electricity but rather as a joint production of both electricity and carbon dioxide. However, to assure an economist that actual change has occurred, a change in the "price situation" is needed. We think that is changing as well. We have witnessed various approaches in the pricing of the "waste product." For example, we have witnessed the pricing of carbon dioxide (Newell et al.[52]; Carl & Fedor [6]).

In our interpretation, the pricing of the waste products corresponds to Piquovian taxation as first suggested by Arthur Pigou [57]. In this approach, the meaning of taxation is not merely to collect public revenues but to guide waste producers toward socially acceptable choices. In the thesis, we clarify in numerous places what is meant by socially acceptable choices. Even more extensively, we concentrate on the details of waste pricing: on direct and indirect ways of pricing. The price can be implemented directly by setting a tax on waste, or the policy can set the price only indirectly by imposing a quota on waste and establishing a market where the quota is traded. These two implementations—environmental taxes and tradable permits—are called market-based instruments in the literature (Stavins [74], [75]). In considering their mutual relationship, we regard the paper of Weitzman [84] as a seminal study. Weitzman shows that it matters whether the policy directly controls the price (the tax) or the quantity (the quota) especially when regulation suffers from uncertainties.

We complement the study of instrument choice by focusing on discrete pollution choices and instrument payments. We base our theoretical framework on a simple observation that the instrument payments guide the polluting units in their discrete choices. The market-based policies set a unit price to an unpriced commodity but they simultaneously create transfer payments. We study how the payments influence the discrete production choices like whether to close down the operation for good or not, whether to voluntarily participate in regulation or to stay outside of it, whether to modify the existing production line, to invest to a new green factory, or to produce in an old style with the same technology as before the regulation. Importantly, discrete choices are major choices, as a single discrete choice typically reduces several units of waste in a cost-effective way.

We assume that a transfer payment contains two parts. The first part is the unit price of emissions times the amount of emissions. As such, it can be interpreted as a factor payment (for the use of environment). The second part is called a subsidy, which offers some flexibility in regulation. A central characteristic of a subsidy is the conditionality of the payment. It is paid to a producing unit only if it satisfies a certain condition (e.g., it is an active firm, uses a certain kind of technology, or participates in a public program). In every case, both parts in a transfer payment are assumed to depend on the unit price of emissions. Consequently, the policy-maker can directly control the tax payments while it controls the quota payments only indirectly. This is the central distinction in the thesis.

We study our research question in three cases. First, we study direct political motives behind subsidization. We study subsidization of firms in the fear of firm closures and in the hope for green investments. In both cases, the environmental agency sees no reason for subsidization but rather take it as a constraint in policy-making. In our second case, the agency accepts subsidization, as the subsidies are vital in correcting the weaknesses of existing regulation. Without subsidization, the participation in regulation is only imperfect, as the mandatory regulation covers only part of the relevant polluters. In the third case, the social welfare again calls for subsidization because of positive externalities. Investments in green technology in one sector enhances the productivity of green technology in another sector. Without proper subsidization in the externality-generating sector, its firms do not properly invest in green technology.

We find three kinds of effects in instrument choice: base, cost, and volume effects. Base effects catch influences that exist within the polluting industry. In Chapter 2, the voluntary provision succeeds in attracting cost-reducing projects under the regulation. This is further reflected in the downward shift of the aggregate cost curve. Likewise, in Chapter 3, a knowledge spillover is reflected in the cost curve. We assume that the spillover operates within the green technology. Consequently, the aggregate emission cost curve shifts downwards as some polluting units in the externality-receiving sector benefit from the positive externality. We show how base effects end up favoring the quantity instrument in the instrument choice. A decrease in the costs of regulation inside the polluting industry allows the regulation to emphasize the emission damages, so a decrease allows the regulation to emphasize the stability of emissions.

The cost effect records inefficiency in the instrument choice. Cost effects evolve as subsidization induces inefficiency. We show how the cost effect favors stability in the unit price of emissions. As an emission tax is an extremely stable price, the cost effect is very favorable to taxation. Taken together, base and cost effects are found to pull in opposite directions.

Base and cost effects are all intuitively clear and well behaving. Their influences are monotonic in the sense that they favor only one instrument. Our analyses indicate that volume effects are less predictable, but there is no difficulty in defining the effect. It is the effect that arises when the tradable emission quota is no longer fixed. However, there are regimes where the volume effect favors the quantity instrument and regimes where it does not favor. We derive these regimes as a function of the various parameters of the model. In our simple framework of Chapter 1, we study at length how various subsidies affect the nature of the volume effect.

Overall, we provide a simple framework for the study of subsidized instrument choice. In doing that, we use some simplifying assumptions. In particular, we concentrate on the determination of two endogenous variables within the regulated industry. Furthermore, in studying the discrete choices, we assume that the polluting units take the level of their activity as given.

Let us briefly discuss these simplifying assumptions. In the thesis, a polluting sector is divided into two polluting sectors that are further divided into two production technologies. These assumptions together allow the simultaneous determination of four choice variables. Even though we mainly concentrate on the determination of two variables, we outline an extended model at the end of Chapter 1 that determines four outcomes. We think that further research using extended models is needed, as the (two-variable) model actually analyzed may produce too specific results.

There is another, perhaps more subtle, concern. An important difference between the models in Chapter 1 and 2 is that Chapter 2 studies intra-firm permit trading while Chapter 1 does not study. Intra-firm permit trading occurs if some polluting units have permit surpluses, so that they can sell permits in the permit markets. We find that instrument choice is very sensitive to the fact whether intra-firm permit trading occurs or not. More precisely, we find that the volume effect is very sensitive to the various assumptions about intra-firm permit trading. However, our extended model at the end of Chapter 1 allows permit trades between different technologies, so it promises an opportunity to study further this particular question. Overall, we conclude that further research on the volume effect is evidently needed.

The second simplifying assumption in the thesis concerns the unit-level choices. The choices are simple, since we only take the type of the technology as a choice variable but omit the level of emissions.[19] It is possible that our models may (again) produce outcomes that may be too specific. In particular, the models may yield more efficient allocations than extended models would do.[20] These extended models assume both discrete and continuous emission choices and they typically find that only zero subsidies are efficient. However, it is important to note that we are primarily interested not in efficient but inefficient outcomes, since we are interested in instrument choices under inefficient implementations. In fact, we show in many occasions that an efficient implementation produces the original comparative advantage of Weitzman [84]. In addition, even though the extended models may offer a more realistic view, they are more complex as well. The complexity accumulates as we already study second-best outcomes in the thesis.[21]

We stress once more that the discrete emission reductions are the basis of the thesis. We see that the discrete choices are the major means in improving the quality

[19] In this context, we regard Amacher and Malik [2] as an example of an extended model.

[20] We base this view on our own calculations and in models like Spulber [71] and Kling and Zhao [30].

[21] That is, the original Weitzman study [84] is a second-best study (Phaneuf and Requate [56]).

of air, land, and water. In formulating this view, our models apply rather rude approximations as they simply ignore the continuous choices. In the future, we would like to be more explicit about the modeling issue. We would like to evaluate the error that the approximation induces. Furthermore, we could develop new approximations where both continuous and discrete choices are included, the continuous choices are of secondary importance, and the linear-quadratic framework of Weitzman [84] applies.

The extensions to our framework will surely expand our understanding of the various effects in the implementation of subsidized market-based instruments. However, the current study has notable merits, too. It provides the first systematic framework for the comparative study of subsidized market-based instruments. In particular, we like to note that we consider the main qualitative findings robust. That is, even if we study discrete choices in an extended subsidized framework, the cost, the volume, and the base effects will exist even though their mutual relations may be affected.

BIBLIOGRAPHY

[1] Adar, Z., & Griffin, J. M. (1976). Uncertainty and the choice of pollution control instruments. *Journal of Environmental Economics and Management*, *3(3)*, 178-188.

[2] Amacher, G. S., & Malik, A. S. (2002). Pollution taxes when firms choose technologies. *Southern Economic Journal*, 891-906.

[3] Baldursson, F. M., & Von Der Fehr, N. H. M. (2004). Prices vs. quantities: the irrelevance of irreversibility. *Scandinavian Journal of Economics*, *106(4)*, 805-821.

[4] Baumol, W. J., & Oates, W. E. (1988). *The theory of environmental policy*. Cambridge University Press.

[5] Bento, A. M., Kanbur, R., & Leard, B. (2015). Designing efficient markets for carbon offsets with distributional constraints. *Journal of Environmental Economics and Management*, *70*, 51-71.

[6] Carl, J., & Fedor, D. (2016). Tracking global carbon revenues: A survey of carbon taxes versus cap-and-trade in the real world. *Energy Policy*, *96*, 50-77.

[7] Chávez, C., & Stranlund, J. K. (2009). A note on emissions taxes and incomplete information. *Environmental and resource economics*, *44(1)*, 137-144.

[8] Collinge, R. A., & Oates, W. E. (1982). Efficiency in pollution control in the short and long runs: A system of rental emission permits. *The Canadian Journal of Economics/Revue canadienne d'Economique, 15(2)*, 346-354.

[9] Cramton, P., & Kerr, S. (2002). Tradeable carbon permit auctions: How and why to auction not grandfather. *Energy policy*, *30(4)*, 333-345.

[10] D'Amato, A., & Dijkstra, B. R. (2015). Technology choice and environmental regulation under asymmetric information. *Resource and Energy Economics*, *41*, 224-247.

[11] Devlin, R. A., & Grafton, R. Q. (1998). Economic rights and environmental wrongs: property rights for the common good. CW Henderson Publisher.

[12] European Environmental Agency (2018). Atmospheric greenhouse gas concentrations. Retrieved from https://www.eea.europa.eu/data-and-maps/indicators/atmospheric-greenhouse-gas-concentrations-4/assessment

[13] Ellerman, A. D., Joskow, P. L., Schmalensee, R., Bailey, E. M., & Montero, J. P. (2000). Markets for clean air: The US acid rain program. Cambridge University Press.

[14] Ellerman, A. D. (2005). A note on tradeable permits. *Environmental and Resource Economics*, *31(2)*, 123-131.

[15] Ellerman, A. D. (2008). New entrant and closure provisions: How do they distort?. *The Energy Journal*, 63-76.

[16] Ellerman, A. D., Marcantonini, C., & Zaklan, A. (2016). The European union emissions trading system: ten years and counting. *Review of Environmental Economics and Policy*, *10(1)*, 89-107.

[17] European Commission. EU Emissions Trading System (EU ETS) (2018). Retrieved from https://ec.europa.eu/clima/policies/ets_en

[18] Farrow, S. (1995). The dual political economy of taxes and tradable permits. *Economics Letters*, *49(2)*, 217-220.

[19] Frisch, R. (1965). *Theory of production*. D. Reidel Publishing Company

[20] Goodkind, A. L., & Coggins, J. S. (2015). The Weitzman price corner. *Journal of Environmental Economics and Management*, *73*, 1-12.

[21] Griliches, Z. (1957). Hybrid corn: An exploration in the economics of technological change. *Econometrica*, *Journal of the Econometric Society*, 501-522.

[22] Griliches Z. (1992) . *Scandinavian Journal of Economics*. 1992 Supplement, Vol. 94, S29-S47.

[23] Heal, G., & Tarui, N. (2010). Investment and emission control under technology and pollution externalities. *Resource and Energy Economics*, *32(1)*, 1-14.

[24] The Intergovernmental Panel on Climate Change (IPCC (2018). 2007: Working Group III: Mitigation of Climate Change. Retrieved from http://www.ipcc.ch/publications_and_data/ar4/wg3/en/ch11s11-7-2.html

[25] Jaffe, A. B., Newell, R. G., & Stavins, R. N. (2005). A tale of twomarket failures: Technology and environmental policy. *Ecological Economics*, *54(2)*, 164–174.

[26] Jones, C. A., & Scotchmer, S. (1990). The social cost of uniform regulatory standards in a hierarchical government. *Journal of Environmental Economics and Management*, *19(1)*, 61-72.

[27] Joskow, P. L., & Schmalensee, R. (1998). The political economy of market-based environmental policy: the US acid rain program. *The journal of law and economics*, *41(1)*, 37-84.

[28] Kato, K. (2006). Can allowing to trade permits enhance welfare in mixed oligopoly?. *Journal of Economics*, 88(3), 263-283.

[29] Kato, K. (2011). Emission quota versus emission tax in a mixed duopoly. *Environmental Economics and Policy Studies*, 13(1), 43-63.

[30] Kling, C. L., & Zhao, J. (2000). On the long-run efficiency of auctioned vs. free permits. *Economics letters*, *69(2)*, 235-238.

[31] Keohane, N. O. (2009). Cap and trade, rehabilitated: Using tradable permits to control US greenhouse gases. *Review of Environmental Economics and policy*, *3(1)*, 42-62.

[32] Keohane, N. O., Revesz, R. L., & Stavins, R. N. (1998). The choice of regulatory instruments in environmental policy. *Harv. Envtl. L. Rev.*, *22*, 313.

[33] Krysiak, F. C., & Oberauner, I. M. (2010). Environmental policy à la carte: Letting firms choose their regulation. *Journal of Environmental Economics and Management*, *60(3)*, 221-232.

[34] Krysiak, F. C. (2008). Prices vs. quantities: The effects on technology choice. *Journal of Public Economics*, *92(5)*, 1275-1287.

[35] Land Trasport Authority (2018). Certificate of Entitlement (COE). Retrieved from https://www.lta.gov.sg/content/ltaweb/en/roads-and-motoring/owning-a-vehicle/vehicle-quota-system/certificate-of-entitlement-coe.html

[36] Lee, S. H. (1996). An optional permit system for global pollution control. *Economics letters*, 50(1), 79-84.

[37] Malik, Arun S. "Enforcement costs and the choice of policy instruments for controlling pollution." *Economic Inquiry 30.4* (1992): 714-721.

[38] Lewis, T. R. (1996). Protecting the environment when costs and benefits are privately known. *The RAND Journal of Economics*, 819-847.

[39] Malueg, D. A. (1990). Welfare consequences of emission credit trading programs. *Journal of Environmental Economics and management*, 18(1), 66-77.

[40] Mandell, S. (2008). Optimal mix of emissions taxes and cap-and-trade. *Journal of environmental economics and management*, *56(2)*, 131-140.

[41] McKitrick, R., & Collinge, R. A. (2000). Linear Pigovian taxes and the optimal size of a polluting industry. *Canadian Journal of Economics/Revue canadienne d'économique*, *33(4)*, 1106-1119.

[42] Mendelsohn, R. (1984). Endogenous technical change and environmental regulation. *Journal of Environmental Economics and Management*, *11(3)*, 202-207.

[43] Meunier, G. (2018). Prices versus quantities in the presence of a second, unpriced, externality. *Journal of Public Economic Theory*, *20(2)*, 218-238.

[44] Mideksa, T. B, & Weitzman, M. L. (2019). Prices versus Quantities across Jurisdictions. *Journal of the Association of Environmental and Resource Economists 6*, no. 5, 883-891.

[45] Miyamoto, T. (2014). Taxes versus quotas in lobbying by a polluting industry with private information on abatement costs. *Resource and Energy Economics*, *38*, 141-167.

[46] Montero, J. P. (1999) Prices vs. Quantities with Incomplete Enforcement. *Working Paper*.

[47] Montero, J. P. (2000). Optimal design of a phase-in emissions trading program. *Journal of Public Economics*, *75(2)*, 273-291.

[48] Montero, J. P. (2001). Multipollutant markets. *RAND Journal of Economics*, 762-774.

[49] Montero, J. P. (2002). Prices versus quantities with incomplete enforcement. *Journal of Public Economics*, *85(3)*, 435-454.

[50] Montero, J. P. (2008). A simple auction mechanism for the optimal allocation of the commons. *American Economic Review*, *98(1)*, 496-518.

[51] Montero, J. P. (2011). A note on environmental policy and innovation when governments cannot commit. *Energy Economics*, 33, S13-S19.

[52] Newell, R. G., Pizer, W. A., & Raimi, D. (2013). Carbon markets 15 years after Kyoto: Lessons learned, new challenges. *The Journal of Economic Perspectives*, *27(1)*, 123-146.

[53] Nikula H. (2014) Environmental policy implementation under structural break. Unpublished working paper.

[54] Pezzey, J. C. (2003). Emission taxes and tradeable permits a comparison of views on long-run efficiency. *Environmental and Resource Economics*, *26(2)*, 329-342.

[55] Pezzey, J. (1992). The symmetry between controlling pollution by price and controlling it by quantity. *Canadian Journal of Economics*, 983-991.

[56] Phaneuf, D. J., & Requate, T. (2016). *A course in environmental economics: theory, policy, and practice*. Cambridge University Press.

[57] Pigou, A. (2017). *The economics of welfare*. Routledge.

[58] Polinsky, A. M., & Shavell, S. (1982). Pigouvian taxation with administrative costs. *Journal of Public Economics*, 19(3), 385-394.

[59] Quirion, P. (2004). Prices versus quuantities in a second-best setting. *Environmental and Resource Economics*, *29*, 337-360.

[60] Requate, T. (1993). Pollution control in a Cournot duopoly via taxes or permits. *Journal of Economics*, 58(3), 255-291.

[61] Requate, T. (2005). Dynamic incentives by environmental policy instruments—a survey. *Ecological economics*, *54(2-3)*, 175-195.

[62] Roberts, M. J., & Spence, M. (1976). Effluent charges and licenses under uncertainty. *Journal of Public Economics*, *5(3)*, 193-208.

[63] Samuelson, P. A. (1954). The pure theory of public expenditure. *The Review of Economics and Statistics*, 387-389.

[64] Sandmo, A. (2011). Economics evolving: *A history of economic thought*. Princeton University Press.

[65] Sartzetakis, E. S. (1997). Tradeable emission permits regulations in the presence of imperfectly competitive product markets: welfare implications. *Environmental and Resource Economics*, 9(1), 65-81.

[66] Schmalensee, Richard, and Robert N. Stavins. 2013. "The SO2 Allowance Trading System: The Ironic History of a Grand Policy Experiment." *Journal of Economic Perspectives,* Volume 27, Number 1, Winter, pp. 103-122.

[67] Schmalensee, R., & Stavins, R. (2015). Lessons learned from three decades of experience with cap-and-trade (*No. w21742*). National Bureau of Economic Research.

[68] Shinkuma, T., & Sugeta, H. (2016). Tax versus emissions trading scheme in the long run. *Journal of Environmental Economics and Management*, *75*, 12-24.

[69] Schöb, R. (1996). Choosing the right instrument. *Environmental and Resource Economics*, *8*, 399-416.

[70] Smulders, S., & Di Maria, C. (2012). The cost of environmental policy under induced technical change. *Cesifo Working Paper*, No. 3886

[71] Spulber, D. F. (1985). Effluent regulation and long-run optimality. *Journal of Environmental Economics and Management*, *12(2)*, 103-116.

[72] Stavins, R. N. (1995). Transaction costs and tradeable permits. *Journal of environmental economics and management*, 29(2), 133-148.

[73] Stavins, R. N. (1996). Correlated uncertainty and policy instrument choice. *Journal of environmental economics and management*, *30(2)*, 218-232.

[74] Stavins, R. N. (2007). *Environmental economics* (No. w13574). National Bureau of Economic Research.

[75] Stavins, R. N. (2010). Market-based environmental policies. In *Public policies for environmental protection* (pp. 41-86). Routledge.

[76] Stern, N., Peters, S., Bakhshi, V., Bowen, A., Cameron, C., Catovsky, S., ... & Edmonson, N. (2006). Stern Review: The economics of climate change (Vol. 30, p. 2006). London: HM treasury.

[77] Stern, N., & Taylor, C. (2007). Climate change: Risk, ethics, and the Stern review. Science, 317(5835), 203-204.

[78] Stranlund, J. K., & Dhanda, K. K. (1999). Endogenous monitoring and enforcement of a transferable emissions permit system. *Journal of Environmental Economics and Management*, 38(3), 267-282.

[79] Stranlund, J. K., Chavez, C. A., & Villena, M. G. (2009). The optimal pricing of pollution when enforcement is costly. *Journal of Environmental Economics and Management*, 58(2), 183-191.

[80] Tietenberg, T. H., & Lewis, L. (2016). *Environmental and natural resource economics*. Routledge.

[81] Tol, R. S., & Yohe, G. W. (2009). The stern review: a deconstruction. *Energy policy*, *37*(3), 1032-1040.

[82] U.S. Environmental Protection Agency (2018). Acid Rain Program. Retrieved from https://www.epa.gov/airmarkets/acid-rain-program

[83] Wagner, G., & Weitzman, M. L. (2015). *Climate shock: The economic consequences of a hotter planet*. Princeton University Press.

[84] Weitzman, M. L. (1974). Prices vs. quantities. *The review of economic studies*, *41(4)*, 477-491.

[85] Weitzman, M. L. (2007). A review of the Stern Review on the economics of climate change. *Journal of economic literature*, 45(3), 703-724.

[86] Weitzman, M. L. (2014). Can negotiating a uniform carbon price help to internalize the global warming externality?. *Journal of the Association of Environmental and Resource Economists*, 1(1/2), 29-49.

[87] Weitzman, M. L. (2017a). Voting on prices vs. voting on quantities in a World Climate Assembly. *Research in Economics*, 71(2), 199-211.

[88] Weitzman, M. L. (2017b). On a world climate assembly and the social cost of carbon. *Economica*, 84(336), 559-586.

[89] Weitzman, M. L. (2018). Prices or quantities dominate banking and borrowing (No. w24218). National Bureau of Economic Research.

[90] Wirl, F. (2014). Taxes versus permits as incentive for the intertemporal supply of a clean technology by a monopoly. *Resource and Energy Economics*, 36(1), 248-269.

[91] Woerdman, E. (2015). The EU greenhouse gas emissions trading scheme. In E. Woerdman *et, al*. (Eds.). Essential EU climate law. Edward Elgar Publishing.

[92] Yates, A. J., & Cronshaw, M. B. (2001). Pollution permit markets with intertemporal trading and asymmetric information. *Journal of Environmental Economics and Management*, *42(1)*, 104-118.

[93] Åhman, M., Burtraw, D., Kruger, J., & Zetterberg, L. (2007). A Ten-Year Rule to guide the allocation of EU emission allowances. *Energy Policy*, *35(3)*, 1718-173

A INEFFICIENT SUBSIDIZATION

A.1 Expected Welfare Maximization

We start by deriving the optimal societal policy of Section 1.3.2.1. By definition, the policy consists of an optimal emission price (τ) and sector specific thresholds (l_0, l_1). The objective is as follows:

$$\underset{\tau,l_0,l_1}{Max}\ EW = E\left[\int_0^{\lambda_0(\tau,l_0,\theta)} B_0 d\lambda + \int_0^{\lambda_1(\tau,l_1,\theta)} B_1 d\lambda\right] - ED(e(\lambda_0(\tau,l_0,\theta),\lambda_1(\tau,l_1,\theta))$$

such that

$$e = \lambda_0(\tau,l_0,\theta)\alpha_0 + \lambda_1(\tau,l_1,\theta)\alpha_1.$$

The first order conditions are

$$\frac{dEW}{d\tau} = E\left[\big(B_0(\lambda_0(\tau,l_0,\theta)) - \alpha_0 D'(e)\big)\frac{d\lambda_0(\tau,l_0,\theta)}{d\tau}\right] + E\left[\big(B_1(\lambda_1(\tau,l_1,\theta)) - \alpha_1 D'(e)\big)\frac{d\lambda_1(\tau,l_1,\theta)}{d\tau}\right] = 0, \tag{A.1}$$

$$\frac{dEW}{dl_0} = E\left[\big(B_0(\lambda_0(\tau,l_0,\theta)) - \alpha_0 D'(e)\big)\frac{d\lambda_0(\tau,l_0,\theta)}{dl_0}\right] = 0, \tag{A.2}$$

and

$$\frac{dEW}{dl_1} = E\left[\big(B_1(\lambda_1(\tau,l_1,\theta)) - \alpha_1 D'(e)\big)\frac{d\lambda_1(\tau,l_1,\theta)}{dl_1}\right] = 0. \tag{A.3}$$

In particular, by Equations (1.1) and (1.4) in the main text, we have

$$B_i(\lambda_i(\tau, l_i, \theta)) = \tau(\alpha_i - l_i),$$

where $i = 0, 1$. As far as $\frac{d\lambda_1(\tau, l_1, \theta)}{d\tau} \neq 0$, then (by Equations (A.2) and (A.3))

$$\tau(\alpha_i - l_i) - \alpha_1 E\left[D'(e)\right] = 0,$$

where $i = 0, 1$. Furthermore, Equation (A.1) can be rewritten as

$$\begin{aligned}
&E\left[\left(\tau(\alpha_0 - l_0) - \alpha_0 D'(e)\left(-\frac{\alpha_0 - l_0}{c_0}\right)\right]\\
&+E\left[\left(\tau(\alpha_1 - l_1) - \alpha_1 D'(e)\alpha_1\left(-\frac{\alpha_1 - l_1}{c_1}\right)\right]\\
&= E\left[-\tau\left(\frac{(\alpha_0 - l_0)^2}{c_0} + \frac{(\alpha_0 - l_0)^2}{c_0}\right)\right]\\
&+E\left[\left(\alpha_0\frac{\alpha_0 - l_0}{c_0} + \alpha_1\frac{\alpha_1 - l_1}{c_1}\right)D'(e)\right] = 0,
\end{aligned}$$

or, by the definitions in Equations (1.12) and (1.17), as

$$-\frac{\tau}{\gamma^l} + \frac{E\left[D'(e)\right]}{\gamma^L} = 0.$$

Note that we can also consider the tax rate as a redundant policy variable. That is, if Equations (A.2) and (A.3) hold simultaneously, then Equation (A.1) holds as well. At the implementation stage, the units should be charged payments equal to S_0 and S_1 in sectors zero and one, respectively. These payments should satisfy $S_0 = \alpha_0 D'(e)$ and $S_1 = \alpha_1 D'(e)$, i.e., they are equal to efficient tax payments in a market-based implementation.

A.2 Optimal Policies Yield Only Nominal Differences

We claim that the different optimal tax rates (introduced in Section 1.3.2.1) yield identical level of emissions *ex-post*. To see this, write first the emissions (Equation (1.26)) as

$$
\begin{aligned}
e(s,\theta) &= \alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)-\frac{c_1(\alpha_0-l_0)\alpha_0+c_0(\alpha_1-l_1)\alpha_1}{c_0c_1}s \\
&= \alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)-\frac{c_1\left(\frac{\alpha_0-l_0}{\alpha_0}\right)\alpha_0^2+c_0\left(\frac{\alpha_1-l_1}{\alpha_1}\right)\alpha_1^2}{c_0c_1}s,
\end{aligned}
$$

or, by the efficiency rule (Equation (1.24)), as

$$
\begin{aligned}
e(s,\theta) &= \alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)-\omega\frac{c_1\alpha_0^2+c_0\alpha_1^2}{c_0c_1}s \\
&= \alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)-\omega\frac{s}{\gamma},
\end{aligned}
\tag{A.4}
$$

where γ is the slope of the marginal abatement function. In the first implementation, $l_0=l_1=0$, $\omega=1$, and $s=\tau^0$, so

$$
e(\tau^0,\theta)=\alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)-\frac{\tau^0}{\gamma}.
$$

Alternatively, $l_0>0$, $l_1>0$, $0<\omega<1$, and $s=\tau^e=\frac{\tau^0}{\omega}$, so

$$
\begin{aligned}
e(\tau^e,\theta) &= \alpha_0\left(\frac{b_0+\theta}{c_0}\right)+\alpha_1\left(\frac{b_1+\theta}{c_1}\right)-\omega\frac{\tau^e}{\gamma} \\
&= \alpha_0\frac{b_0}{c_0}+\alpha_1\frac{b_1}{c_1}-\frac{\tau^0}{\gamma}.
\end{aligned}
$$

Clearly, $e(\tau^0,\theta)=e(\tau^e,\theta)$.

A.3 Optimal Permit Policies

We study next implementations of the optimal societal policy by tradable permits (see Section 1.3.2.2). Let us study first the supply of aggregate permits (L) in the sterilized system. By Equations (1.12) and (1.14), we write the price as

$$\overline{p}^L = \frac{c_0 c_1}{c_1(\alpha_0 - l_0)\alpha_0 + c_0(\alpha_1 - l_1)\alpha_1}\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - L\right)$$
$$= \frac{c_0 c_1}{c_1\left(\frac{\alpha_0 - l_0}{\alpha_0}\right)\alpha_0^2 + c_0\left(\frac{\alpha_1 - l_1}{\alpha_1}\right)\alpha_1^2}\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - L\right)$$

or, after incorporating the efficiency rule (Equation (1.24)), as

$$\overline{p}^L = \frac{\gamma}{\omega}\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - L\right),$$

where is γ is the slope of the marginal abatement function. We set first $l_0 = l_1 = 0$. As $\omega = 1$, we have

$$\overline{p}^L = p^0 = \gamma\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - L\right).$$

Alternatively, we set $l_0 > 0$ and $l_1 > 0$, so $0 < \omega < 1$ and

$$\overline{p}^L = p^e = \frac{\gamma}{\omega}\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - L\right). \tag{A.5}$$

It then holds that

$$p^e = \frac{p^0}{\omega}.$$

Let us study next the supply of auctioned permits (l) in the non-sterilized system. Following the steps above (together with Equations (1.17) and (1.19)), we may write the price as

$$\overline{p}^l = \frac{\gamma}{\omega^2}\left(\omega\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1}\right) - l\right). \tag{A.6}$$

In particular, if we set $l_0 = l_1 = 0$ and $l = L$, then

$$\overline{p}^l = p^0 = \gamma\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - L\right).$$

On the other hand, if $l_0 > 0$ and $l_1 > 0$, then $0 < \omega < 1$. As optimality requires that $\overline{p}^l = \tau^e$, we incorporate condition $l = \omega L$ into Equation (A.6). Consequently,

$$\overline{p}^l = p^e = \frac{\gamma}{\omega}\left(\frac{b_0\alpha_0}{c_0} + \frac{b_1\alpha_1}{c_1} - L\right).$$

In summary, it holds between optimal implementations that $l = \omega L \leq L$.

A.4 Characteristics of Optimal Policy

Regarding the second-best policy in Section 1.3.3, we state:

i. By assuming efficiency, the second-best price and quantity becomes first-best.
ii. $\tau^s > \tau^0$.
ii. $e^s > e^*$.

Proof:

i. We write that (see Equations (1.12) and (1.17))

$$\gamma^l = \frac{c_0 c_1}{c_1(\alpha_0 - l_0)^2 + c_0(\alpha_1 - l_1)^2} = \frac{c_0 c_1}{c_1\left(\frac{\alpha_0 - l_0}{\alpha_0}\right)^2(\alpha_0)^2 + c_0\left(\frac{\alpha_1 - l_1}{\alpha_1}\right)^2(\alpha_1)^2}$$

and

$$\gamma^L = \frac{c_0 c_1}{c_1(\alpha_0 - l_0)\alpha_0 + c_0(\alpha_1 - l_1)\alpha_1} = \frac{c_0 c_1}{c_1\left(\frac{\alpha_0 - l_0}{\alpha_0}\right)(\alpha_0)^2 + c_0\left(\frac{\alpha_1 - l_1}{\alpha_1}\right)(\alpha_1)^2},$$

or, after applying the efficiency condition

$$\frac{\alpha_0 - l_0}{\alpha_0} \equiv \frac{\alpha_1 - l_1}{\alpha_1} \equiv \omega,$$

we write that

$$\gamma^l = \frac{1}{\omega^2} \frac{c_0 c_1}{c_1(\alpha_0)^2 + c_0(\alpha_1)^2} = \frac{\gamma}{\omega^2}$$

and

$$\gamma^L = \frac{1}{\omega} \frac{c_0 c_1}{c_1(\alpha_0)^2 + c_0(\alpha_1)^2} = \frac{\gamma}{\omega}.$$

Then, the second-best policy targets under efficiency become

$$
\begin{aligned}
\tau^s &= \frac{d\gamma^L\gamma^l}{[\gamma^L]^2 + d\gamma^l}\left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1}\right) = \frac{d\frac{\gamma}{\omega^3}}{\frac{\gamma}{\omega^2} + \frac{d}{\omega^2}}\left(\alpha_0\left(\frac{b_0}{c_0}\right) + \alpha_1\left(\frac{b_1}{c_1}\right)\right) \\
&= \frac{1}{\omega}\frac{d\gamma}{\gamma + d}\left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1}\right) = \frac{\tau^0}{\omega} = \tau^e
\end{aligned}
$$

and

$$
\begin{aligned}
e^s &= \left(\frac{[\gamma^L]^2}{[\gamma^L]^2 + d\gamma^l}\right)\left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1}\right) = \frac{\frac{\gamma^2}{\omega^2}}{\frac{\gamma^2}{\omega^2} + \frac{d\gamma}{\omega^2}}\left(\alpha_0\left(\frac{b_0}{c_0}\right) + \alpha_1\left(\frac{b_1}{c_1}\right)\right) \\
&= \left(\frac{\gamma}{\gamma + d}\right)\left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1}\right) = e^*.
\end{aligned}
$$

ii. By Equations (1.36) and (1.38), we write the difference between tax rates as

$$
\begin{aligned}
\tau^s - \tau^0 &= \left(\frac{d\gamma^L\gamma^l}{[\gamma^L]^2 + d\gamma^l} - \frac{\gamma d}{\gamma + d}\right)\left(\alpha_0\frac{b_0}{c_0} + \alpha_1\frac{b_1}{c_1}\right) \\
&= d\gamma\left(\frac{(F_U - \rho)\gamma + (F_U - 1)d}{(\gamma\rho + d)(\gamma + d)}\right)\left(\alpha_0\frac{b_0}{c_0} + \alpha_1\frac{b_1}{c_1}\right),
\end{aligned}
$$

where

$$
F_U = \frac{\gamma^L}{\gamma} = \frac{c_1(\alpha_0)\alpha_0 + c_0(\alpha_1)\alpha_1}{c_1(\alpha_0 - l_0)\alpha_0 + c_0(\alpha_1 - l_1)\alpha_1} > 1
$$

and

$$
\rho = \frac{[\gamma^L]^2}{\gamma\gamma^l}.
$$

If we further denote

$$
0 < F_M = \frac{\gamma^L}{\gamma^l} = \frac{c_1(\alpha_0 - l_0)^2 + c_0(\alpha_1 - l_1)^2}{c_1(\alpha_0 - l_0)\alpha_0 + c_0(\alpha_1 - l_1)\alpha_1} < 1,
$$

then it holds that

$$
F_U - \rho = F_U(1 - F_M).
$$

Consequently, we know that $F_U > \rho$, so

$$
\tau - \tau^0 > 0.
$$

iii. By Equations (1.33) and (1.37), we write the difference between emissions as

$$e^s - e^* = \left(\frac{[\gamma^L]^2}{[\gamma^L]^2 + d\gamma^l} - \frac{\gamma}{\gamma + d} \right) \left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1} \right).$$

By further manipulations, it holds that

$$\begin{aligned}
e^s - e^* &= \frac{\gamma^l}{\gamma^l} \left(\frac{\gamma \frac{[\gamma^L]^2}{\gamma\gamma^l}}{\gamma \frac{[\gamma^L]^2}{\gamma\gamma^l} + d} - \frac{\gamma}{\gamma + d} \right) \left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1} \right) \\
&= \left(\frac{\gamma\rho}{\gamma\rho + d} - \frac{\gamma}{\gamma + d} \right) \left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1} \right) \\
&= \left(\frac{\gamma\rho d - d\gamma}{(\gamma\rho + d)(\gamma + d)} \right) \left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1} \right) \\
&= \frac{\rho - 1}{(\gamma\rho + d)} \frac{d\gamma}{(\gamma + d)} \left(\frac{\alpha_0 b_0}{c_0} + \frac{\alpha_1 b_1}{c_1} \right).
\end{aligned}$$

We do not derive the magnitude of ρ here. However, it is shown (see Equation (1.58) in the main text) that $\rho > 1$. It then holds that

$$e^s - e^* > 0.$$

A.5 Various Differences in Prices and Quantities

To understand better our upcoming results of comparative advantage, it is helpful first to study the various paths for the emissions and prices. Overall, we have three separate system to study: taxes, sterilized permits, and non-sterilized permits.

A.5.1 Part I

In the first part, we study how the different permit prices evolve as a function of uncertainty. By using various definitions from Section 1.4.1 (Equations (1.45), (1.46), and (1.47)), we calculate that

$$
\begin{aligned}
p^L(\theta) &= \overline{p}^L + \gamma^L\left(\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}\right)\theta \\
&= \overline{p}^L + \frac{c_0 c_1}{c_1(\alpha_0 - l_0)\alpha_0 + c_0(\alpha_1 - l_1)\alpha_1}\left(\frac{c_1\alpha_0 + c_0\alpha_1}{c_0 c_1}\right)\theta \\
&= \overline{p}^L + \frac{c_1\alpha_0}{c_1(\alpha_0 - l_0)\alpha_0}\frac{1 + \frac{c_0\alpha_1}{c_1\alpha_0}}{1 + \frac{c_0(\alpha_1 - l_1)\alpha_1}{c_1(\alpha_0 - l_0)\alpha_0}}\theta = \overline{p}^L + \frac{1}{(\alpha_0 - l_0)}\frac{1 + ua}{1 + uka}\theta
\end{aligned}
$$

and

$$
\begin{aligned}
p^l(\theta) &= \overline{p}^l + \gamma^l\left(\frac{(\alpha_0 - l_0)}{c_0} + \frac{(\alpha_1 - l_1)}{c_1}\right)\theta \\
&= \overline{p}^l + \left(\frac{c_0 c_1}{c_1(\alpha_0 - l_0)^2 + c_0(\alpha_1 - l_1)^2}\right)\left(\frac{(\alpha_0 - l_0)}{c_0} + \frac{(\alpha_1 - l_1)}{c_1}\right)\theta \\
&= \overline{p}^l + \left(\frac{c_1(\alpha_0 - l_0) + (\alpha_1 - l_1)c_0}{c_1(\alpha_0 - l_0)^2 + c_0(\alpha_1 - l_1)^2}\right)\theta \\
&= \overline{p}^l + \frac{c_1(\alpha_0 - l_0)}{c_1(\alpha_0 - l_0)^2}\left(\frac{1 + \frac{(\alpha_1 - l_1)c_0}{c_1(\alpha_0 - l_0)}}{1 + \frac{c_0(\alpha_1 - l_1)^2}{c_1(\alpha_0 - l_0)^2}}\right)\theta = \overline{p}^l + \frac{1}{\alpha_0 - l_0}\frac{1 + uk}{1 + uk^2}\theta.
\end{aligned}
$$

By Section 1.3.3, it holds that

$$
\overline{p}^L = E\left[p^L(\theta)\right] = \overline{p}^l = E\left[p^l(\theta)\right] = \overline{p},
$$

so we further calculate that

$$
\begin{aligned}
&p^L(\theta)-p^l(\theta) \qquad \text{(A.7)}\\
&=\frac{1}{(\alpha_0-l_0)}\frac{1+ua}{1+uka}\theta-\frac{1}{(\alpha_0-l_0)}\left(\frac{1+uk}{1+uk^2}\right)\theta\\
&=\frac{\theta}{(\alpha_0-l_0)}\left(\frac{1+ua}{1+uka}-\frac{1+uk}{1+uk^2}\right)\\
&=\frac{\theta}{(\alpha_0-l_0)}\left(\frac{(1+ua)\left(1+uk^2\right)-(1+uka)(1+uk)}{(1+uka)(1+uk^2)}\right)\\
&=\frac{\theta}{(\alpha_0-l_0)}\frac{u(a-k)(1-k)}{(1+uka)(1+uk^2)}.
\end{aligned}
$$

A.5.2 Part II

In this part, we concentrate on differences in emissions. We have

$$
e(s;\theta)=\alpha_0\frac{b_0}{c_0}+\alpha_1\frac{b_1}{c_1}+\frac{\alpha_0c_1+\alpha_1c_0}{c_0c_1}\theta-\frac{1}{\gamma^L}s,
$$

so

$$
e(p^L(\theta))-e(\tau(\theta))=-\frac{1}{\gamma^L}\left[\overline{p}+\gamma^L\left(\frac{\alpha_0}{c_0}+\frac{\alpha_1}{c_1}\right)\theta\right]+\frac{\overline{p}}{\gamma^L}=-\frac{c_1\alpha_0+c_0\alpha_1}{c_1c_0}\theta.
$$

Likewise,

$$
\begin{aligned}
&e(p^l(\theta))-e(\tau(\theta))=-\frac{\overline{p}}{\gamma^L}+\frac{\overline{p}}{\gamma^L}\\
&-\frac{1}{\gamma^L}\left(\frac{c_0c_1}{c_1(\alpha_0-l_0)^2+c_0(\alpha_1-l_1)^2}\right)\left(\frac{\alpha_0-l_0}{c_0}+\frac{\alpha_1-l_1}{c_1}\right)\theta\\
&=-\frac{1}{\gamma^L}\left(\frac{c_1(\alpha_0-l_0)+c_0(\alpha_1-l_1)}{c_1(\alpha_0-l_0)^2+c_0(\alpha_1-l_1)^2}\right)\theta\\
&=-\frac{1}{\gamma^L(\alpha_0-l_0)}\left(\frac{1+\frac{c_0(\alpha_1-l_1)}{c_1(\alpha_0-l_0)}}{1+\frac{c_0(\alpha_1-l_1)^2}{c_1(\alpha_0-l_0)^2}}\right)\theta=-\frac{1}{\gamma^L}\frac{1}{(\alpha_0-l_0)}\left(\frac{1+uk}{1+uk^2}\right)\theta.
\end{aligned}
$$

Finally,

$$e(p^L(\theta)) - e(p^l(\theta)) = -\frac{p^L(\theta)}{\gamma^L} + \frac{p^l(\theta)}{\gamma^L} = \frac{1}{\gamma^L}\left(p^l(\theta) - p^L(\theta)\right),$$

or, by Equation (A.7) above,

$$e(p^L(\theta)) - e(p^l(\theta)) = \frac{1}{\gamma^L}\frac{(a-k)(k-1)}{\alpha_0 - l_0}\frac{u}{(1+uk^2)(1+uak)}\theta.$$

A.5.3 Part III

Table A.1 depicts different types of emissions paths. It does that within sectors and at industry level. In writing the table, we incorporate cut-off units λ_i (Equations (1.4)) into Equation (1.5) that represents sector-specific emissions. Each category in the table depicts the emissions in the tax regime, in the non-sterilized regime, and in the sterilized regime, respectively. Note that we have incorporated the rule

$$E\left[e(p^L(\theta))\right] = E\left[e(p^l(\theta))\right] = E\left[e(\tau)\right]$$

into the calculations. As a result, each instrument produces the same expected emissions, both within sectors and across polluting industry.

Table A.1 Sector Specific Emissions and Aggregate Emissions Under Various Instruments

Sector 0 Emissions
$e_0(\tau) = \overline{e}_0 + \frac{\alpha_0}{c_0}\theta$ $e_0(p^l) = \overline{e}_0 + \frac{\alpha_0}{c_0}\theta - \frac{\alpha_0}{c_0}(\alpha_0 - l_0)\left(\frac{[R_0(c_1\alpha_0\theta)+R_1(c_0\alpha_1\theta)]}{c_1(\alpha_0-l_0)\alpha_0+c_0(\alpha_1-l_1)\alpha_1}\right)$ $e_0(p^L) = \overline{e}_0 + \frac{\alpha_0}{c_0}\theta - \frac{\alpha_0}{c_0}(\alpha_0 - l_0)\left(\frac{c_1\alpha_0\theta+c_0\alpha_1\theta}{c_1(\alpha_0-l_0)\alpha_0+c_0(\alpha_1-l_1)\alpha_1}\right)$
Sector 1 Emissions
$e_1(\tau) = \overline{e}_1 + \frac{\alpha_1}{c_1}\theta$ $e_1(p^l) = \overline{e}_1 + \frac{\alpha_1}{c_1}\theta - \frac{\alpha_1}{c_1}(\alpha_1 - l_1)\left(\frac{R_0(c_1\alpha_0\theta)+R_1(c_0\alpha_1\theta)}{c_1(\alpha_0-l_0)\alpha_0+c_0(\alpha_1-l_1)\alpha_1}\right)$ $e_1(p^L) = \overline{e}_1 + \frac{\alpha_1}{c_1}\theta - \frac{\alpha_1}{c_1}(\alpha_1 - l_1)\left(\frac{c_1\alpha_0\theta+c_0\alpha_1\theta}{c_1(\alpha_0-l_0)\alpha_0+c_0(\alpha_1-l_1)\alpha_1}\right)$
Industry Emissions
$e(\tau) = \overline{e} + \frac{\alpha_0}{c_0}\theta + \frac{a_1}{c_1}\theta$ $e(p^l) = \overline{e} + \frac{\alpha_0}{c_0}\theta + \frac{\alpha_1}{c_1}\theta - \left(R_0\frac{\alpha_0}{c_0}\theta + R_1\frac{\alpha_1}{c_1}\theta\right)$ $e(p^L) = \overline{e}$

A.6 Comparative Advantage in Terms of Variances

In doing the analysis of comparative advantage, we sometimes operate in terms of variances. This section derives the proper formula of comparative advantage in these cases.

Write the expected benefits in Equation (1.50) as

$$EB(s) = E\left[\frac{(b_0+\theta)^2}{2c_0} + \frac{(b_1+\theta)^2}{2c_1}\right] - \frac{Var(s)}{2\gamma^l} - \frac{(Es)^2}{2\gamma^l}$$

and the expected damages in Equation (1.51) as

$$ED(e(s)) = \frac{d}{2}(Var(e(s))) + \frac{d}{2}(Ee)^2.$$

We have

$$\begin{aligned}\Delta(I,J) &= E\left[(B(I) - D(e(I))) - (B(J) - D(e(J)))\right]\\ &= E\left[(B(I) - B(J)) - (D(e(I)) - D(e(J)))\right].\end{aligned}$$

As it holds that $E(I) = E(J)$ and $E[e(I)] = E[e(J)]$, it then follows that

$$\begin{aligned}\Delta(I,J) &= E\left[\left(-\frac{Var(I)}{2\gamma^l} - \left(-\frac{Var(J)}{2\gamma^l}\right)\right) - \left(\frac{d}{2}Var(e(I)) - \frac{d}{2}Var(e(J))\right)\right]\\ &= \frac{Var(J) - Var(I)}{2}\left[\frac{1}{\gamma^l} - d\left(\frac{Var(e(I)) - Var(e(J))}{Var(J) - Var(I)}\right)\right].\end{aligned}$$

A.7 Uncertain Marginal Damages

In choosing the best instrument, we repeatedly apply an assumption that only the benefits of emissions, not the damages, are uncertain. In this section, we briefly discuss this assumption. In particular, we show that our assumption does not diminish the generality of our results as long as the benefit and damage uncertainties are independent.

To start with, we rewrite the emission damages as

$$D(e(s)) = d\varepsilon e + \frac{d}{2}e^2,$$

where ε is a random variable that shifts the marginal damage curve and

$$E\varepsilon = 0.$$

We have

$$e(\theta) = \alpha_0\left(\frac{b_0+\theta}{c_0}\right) + \alpha_1\left(\frac{b_1+\theta}{c_1}\right) - \frac{s(\theta)}{\gamma^L}$$

and (by Equation (1.44))

$$s(\theta) = \bar{s} + \gamma^L\left(R_0\frac{\alpha_0}{c_0} + R_1\frac{\alpha_1}{c_1}\right)\theta.$$

Then

$$\begin{aligned} E[\varepsilon e] &= E\left[\varepsilon\left(\bar{s} + \gamma^L\left(R_0\frac{\alpha_0}{c_0} + R_1\frac{\alpha_1}{c_1}\right)\theta\right)\right] \\ &= E[\varepsilon\bar{s}] + \gamma^L\left(R_0\frac{\alpha_0}{c_0} + R_1\frac{\alpha_1}{c_1}\right)E[\varepsilon\theta] \\ &= \gamma^L\left(R_0\frac{\alpha_0}{c_0} + R_1\frac{\alpha_1}{c_1}\right)E[\varepsilon\theta]. \end{aligned}$$

As (by assumption)

$$E\varepsilon\theta = 0,$$

then

$$E[\varepsilon e] = 0.$$

Consequently, it holds that

$$ED(e(s)) = \frac{d}{2}E\left[e^2\right].$$

A.8 Characteristics of the Cost and Volume Effects

In discussing the cost effect and volume effect in Section 1.4.3, we claim:

i. $\Theta(I,J) = \Theta(J,I) \equiv \Theta$.

ii. The cost effect favors the instrument with the lower variance in price.

iii. Let $\Theta > 1$ ($\Theta < 1$). The volume effect favors the instrument with the higher (lower) variance in price.

Proof:

i. By Equation (1.55), we have

$$\Theta(I,J) = \left(\gamma^L\right)^2 \frac{Var(e(I)) - Var(e(J))}{Var(J) - Var(I)},$$

so

$$\Theta(J,I) = \left(\gamma^L\right)^2 \frac{Var(e(J)) - Var(e(I))}{Var(I) - Var(J)}.$$

Thus,

$$\begin{aligned}
&\Theta(I,J) - \Theta(J,I) \\
&= \left(\gamma^L\right)^2 \left(\frac{Var(e(I)) - Var(e(J))}{Var(J) - Var(I)} - \frac{Var(e(J)) - Var(e(I))}{Var(I) - Var(J)}\right) \\
&= \left(\gamma^L\right)^2 \frac{(Var(e(I)) - Var(e(J)) + Var(e(J)) - Var(e(I)))}{Var(J) - Var(I)} = 0.
\end{aligned}$$

ii. By Equation (1.52), it holds that

$$\Delta(I,J) = \frac{Var(J) - Var(I)}{2} \left(\frac{1}{\gamma^l} - d\left[\frac{Var(e(I)) - Var(e(J))}{Var(J) - Var(I)}\right]\right).$$

We then calculate that

$$\begin{aligned}\Delta(I,J) &= \frac{Var(J)-Var(I)}{2}\left(\frac{1}{\gamma^l}-d\left[\frac{Var(e(I))-Var(e(J))}{Var(J)-Var(I)}\right]\right) \qquad \text{(A.8)}\\ &= \frac{-[Var(I)-Var(J)]}{2}\left(\frac{1}{\gamma^l}-d\left[\frac{-[Var(e(J))-Var(e(I))]}{-[Var(I)-Var(J)]}\right]\right)\\ &= -\frac{Var(I)-Var(J)}{2}\left(\frac{1}{\gamma^l}-d\left[\frac{Var(e(J))-Var(e(I))}{Var(I)-Var(J)}\right]\right) = -\Delta(J,I).\end{aligned}$$

Alternatively, following the representation in Equation (1.54), we write the comparative advantage as

$$\Delta(I,J) = Var(J)\frac{1-v}{2}\frac{\rho}{(\gamma^L)^2}\left(\gamma - \frac{\Theta}{\rho}d\right).$$

We know that $0 < \frac{1}{\rho} < 1$. Specifically, if $0 < v < 1$, the cost effect favors instrument I, the low-variance instrument. Let $v > 1$ instead. Using Equation (A.8), we have that

$$\begin{aligned}\Delta(J,I) = -\Delta(I,J) &= -\left(Var(J)\frac{1-v}{2}\frac{\rho}{(\gamma^L)^2}\left(\gamma-\frac{\Theta}{\rho}d\right)\right)\\ &= Var(J)\frac{v-1}{2}\frac{\rho}{(\gamma^L)^2}\left(\gamma-\frac{\Theta}{\rho}d\right).\end{aligned}$$

Now, the cost effect favors instrument J, the instrument with the low variance.

iii. The proofs follow the case ii above.

A.9 Cost and Volume Effect Combined

We say that volume effect dominates as long as the size of it exceeds the size of the cost effect. We calculate here the domination region in terms of the model's parameters.

We have the volume effect

$$\Theta(\tau, p^l(\theta)) = 2q-1 = \left(2\frac{1+uk^2}{1+uak}\frac{1+ua}{1+uk}-1\right)$$

and the cost effect

$$\rho = 1 + \frac{u(k-a)^2}{(1+uak)^2}.$$

Calculate first

$$\begin{aligned}
\rho = 1 + \frac{u(k-a)^2}{(1+uak)^2} &= \frac{(1+uak)^2 + u(k-a)^2}{(1+uak)^2} \\
&= \frac{1+2uak+(uak)^2+u\left(k^2-2ak+a^2\right)}{(1+uak)^2} \\
&= \frac{1+uk^2+ua^2\left(1+uk^2\right)}{(1+uak)^2} = \frac{\left(1+uk^2\right)\left(1+ua^2\right)}{(1+uak)^2}.
\end{aligned} \tag{A.9}$$

Thus,

$$\begin{aligned}
&\Theta(\tau, p^l(\theta)) - \rho \\
&= \left(2\frac{1+uk^2}{1+uak}\frac{1+ua}{1+uk} - 1\right) - \left(\frac{\left(1+uk^2\right)\left(1+ua^2\right)}{(1+uak)^2}\right) \\
&= \frac{1+uk^2}{1+uka}\left(2\frac{1+ua}{1+uk} - \frac{1+uka}{1+uk^2} - \frac{1+ua^2}{1+uak}\right) \\
&= \frac{1+uk^2}{1+uka}\left(\left(\frac{1+ua}{1+uk} - \frac{1+uka}{1+uk^2}\right) + \left(\frac{1+ua}{1+uk} - \frac{1+ua^2}{1+uak}\right)\right).
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&\Theta(\tau, p^l(\theta)) - \rho \\
&= \frac{1+uk^2}{1+uka}\left(\frac{ua+uk^2-uka-uk}{(1+uk)(1+uk^2)} + \frac{ua+uak-uk-ua^2}{(1+uk)(1+uak)}\right) \\
&= \frac{1+uk^2}{1+uka}\frac{u}{1+uk}\left(\frac{a(1-k)+k(k-1)}{1+uk^2} + \frac{a+ak-k-a^2}{1+uak}\right) \\
&= \frac{1+uk^2}{1+uka}\frac{u}{1+uk}\left(\frac{(a-k)(1-k)}{1+uk^2} + \frac{(1-a)(a-k)}{1+uak}\right).
\end{aligned}$$

So, we may write that

$$\Theta(\tau, p^l(\theta)) - \rho = \frac{1+uk^2}{1+uka}\frac{u}{1+uk}(a-k)\left(\frac{1-k}{1+uk^2} + \frac{1-a}{1+uak}\right).$$

A.10 Quantities versus Quantities

Our analysis in Section 1.4.8 suggests that the quantity regime can be implemented in two alternative ways. Furthermore, the choice between these implementations is determined by the comparative advantage calculated in Equation (1.76). This measure requires the knowledge of the volume effect Θ and the variance factor v. This section provides formulas for these factors.

In general (see Sections 1.4.2 and 1.4.3), it holds that

$$\Delta(I,J) = Var(J)\frac{1-v}{2}\frac{\rho}{(\gamma^L)^2}\left(\gamma - \frac{\Theta(I,J)}{\rho}d\right), \tag{A.10}$$

where

$$\Theta(I,J) = \left(\gamma^L\right)^2 \frac{Var(e(I)) - Var(e(J))}{Var(J) - Var(I)}$$

and

$$v(I,J) = \frac{Var(I)}{Var(J)}.$$

In Part I, we derive formula for the factor $\Theta(p^l(\theta), p^L(\theta))$, while in Part II, we calculate formula for $v(p^l(\theta), p^L(\theta))$.

A.10.1 Part I

It holds that $Var(e(p^L(\theta))) = 0$, so

$$\Theta(p^l(\theta), p^L(\theta)) = \left(\gamma^L\right)^2 \frac{Var(e(p^l(\theta))}{Var(p^L(\theta)) - Var(p^l(\theta))}.$$

Using the representations in Equations (1.59) and (1.60), we may write that

$$\Theta(p^l(\theta), p^L(\theta)) = \frac{\left(\gamma^L\right)^2 \sigma^2 \left[(1-R_0)\frac{\alpha_0}{c_0} + (1-R_1)\frac{\alpha_1}{c_1}\right]^2}{(\gamma^L)^2 \sigma^2 \left[\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}\right]^2 - (\gamma^L)^2 \sigma^2 \left[R_0(I)\frac{\alpha_0}{c_0} + R_1(I)\frac{\alpha_1}{c_1}\right]^2}.$$

After arranging, it holds that

$$\Theta(p^l(\theta), p^L(\theta)) = \frac{\left[(1-R_0)\frac{\alpha_0}{c_0} + (1-R_1)\frac{\alpha_1}{c_1}\right]^2}{\left[\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}\right]^2 - \left[R_0(I)\frac{\alpha_0}{c_0} + R_1(I)\frac{\alpha_1}{c_1}\right]^2}$$

$$= \frac{\left[(1-R_0)\frac{\alpha_0}{c_0} + (1-R_1)\frac{\alpha_1}{c_1}\right]^2}{\left[\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}\right]^2 - \left[\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1} - (1-R_0)\frac{\alpha_0}{c_0} + (1-R_1)\frac{\alpha_1}{c_1}\right]^2}$$

$$= \frac{1}{2\frac{\left[(1-R_0)\frac{\alpha_0}{c_0} + (1-R_1)\frac{\alpha_1}{c_1}\right]\left[\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}\right]}{\left[(1-R_0)\frac{\alpha_0}{c_0} + (1-R_1)\frac{\alpha_1}{c_1}\right]^2} - 1} = \frac{1}{2\frac{\left[\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}\right]}{\left[(1-R_0)\frac{\alpha_0}{c_0} + (1-R_1)\frac{\alpha_1}{c_1}\right]} - 1}.$$

Using the definitions of R_0 and R_1 (Equations (1.48)), the volume effect becomes

$$\Theta(p^l(\theta), p^L(\theta)) = \frac{1}{2\frac{\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}}{\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1} - R_0\left(\frac{\alpha_0}{c_0} + \frac{k}{a}\frac{\alpha_1}{c_1}\right)} - 1} = \frac{1}{2\frac{1}{(1-R_0)\frac{\frac{\alpha_0}{c_0} + \frac{k}{a}\frac{\alpha_1}{c_1}}{\frac{\alpha_0}{c_0} + \frac{\alpha_1}{c_1}}} - 1}.$$

or, by definitions of u, k and a (Equations (1.47)), it becomes

$$\Theta(p^l(\theta), p^L(\theta)) = \frac{1}{2\frac{1}{1 - \frac{1+uka}{1+uk^2}\frac{1+ku}{1+au}} - 1}.$$

Furthermore, by Equation (1.71), it holds that

$$q(k) = \frac{1+uk^2}{1+uak}\frac{1+ua}{1+uk}, \tag{A.11}$$

so we may finally write that

$$\Theta(p^l(\theta), p^L(\theta)) = \frac{1}{2\frac{1}{1-\frac{1}{q}} - 1} = \frac{q-1}{2q-(q-1)} = \frac{q-1}{q+1}.$$

A.10.2 Part II

Using the same set of definitions as in Part I above, we calculate that

$$
\begin{aligned}
&v(p^l(\theta), p^L(\theta))\\
&= \frac{Var\left(p^l(\theta)\right)}{Var\left(p^L(\theta)\right)} = \left(\frac{\gamma^l\left(\frac{c_1(\alpha_0 - l_0) + c_0(\alpha_1 - l_1)}{c_0 c_1}\right)}{\gamma^L\left(\frac{c_1\alpha_0 + c_0\alpha_1}{c_0 c_1}\right)} \right)^2\\
&= \left(\frac{\frac{c_0 c_1}{c_1(\alpha_0 - l_0)^2 + c_0(\alpha_1 - l_1)^2}}{\frac{c_0 c_1}{c_1(\alpha_0 - l_0)\alpha_0 + c_0(\alpha_1 - l_1)\alpha_1}} \left(\frac{c_1(\alpha_0 - l_0) + c_0(\alpha_1 - l_1)}{c_1\alpha_0 + c_0\alpha_1} \right) \right)^2\\
&= \left(\frac{c_1(\alpha_0 - l_0)\alpha_0}{c_1(\alpha_0 - l_0)^2} \frac{1 + \frac{c_0(\alpha_1 - l_1)\alpha_1}{c_1(\alpha_0 - l_0)\alpha_0}}{1 + \frac{c_0(\alpha_1 - l_1)^2}{c_1(\alpha_0 - l_0)^2}} \frac{c_1(\alpha_0 - l_0)}{c_1\alpha_0} \left(\frac{1 + \frac{c_0(\alpha_1 - l_1)}{c_1(\alpha_0 - l_0)}}{1 + \frac{c_0\alpha_1}{c_1\alpha_0}} \right) \right)^2\\
&= \left(\frac{1 + \frac{c_0(\alpha_1 - l_1)\alpha_1}{c_1(\alpha_0 - l_0)\alpha_0}}{1 + \frac{c_0(\alpha_1 - l_1)^2}{c_1(\alpha_0 - l_0)^2}} \left(\frac{1 + \frac{c_0(\alpha_1 - l_1)}{c_1(\alpha_0 - l_0)}}{1 + \frac{c_0\alpha_1}{c_1\alpha_0}} \right) \right)^2 = \left(\frac{1 + uka}{1 + uk^2} \frac{1 + uk}{1 + ua} \right)^2.
\end{aligned}
$$

So, by Equation (A.11), it holds that

$$
v(p^l(\theta), p^L(\theta)) = \left(\frac{1}{q}\right)^2.
$$

B IMPERFECT PARTICIPATION

B.1 Social Welfares

This section discusses the general welfare properties of various regulatory regimes where participation is either perfect or imperfect and the imperfect participation either includes voluntary participation or does not include it. Under these various regimes, we calculate expected welfare as the difference between benefits and damages of emissions. Furthermore, we will provide optimal regime specific policies, and after inserting the policy variables into welfare functions, we end up with expected maximum value functions.

In general, given an instrument s, we write the social welfare as

$$W(s) = B(s) - D(e(s)).$$

Specifically, in the various regimes to come, the policy will apply the tax rate as an instrument.

B.1.1 Perfect Participation

We study first perfect participation. After setting $l_A = l_0 = l_1 = 0$, we insert the various cut-offs from Equations (2.6), (2.8), and (2.9) into the definition of aggregate benefits (Equation (2.16)). We have

$$B(s) = \frac{(b_A + \theta_A)^2}{2c_A} - \frac{\alpha_A^2}{2c_A}s^2 + \frac{(\Delta b + \Delta\theta)^2}{2\Delta c} - \frac{(\Delta a)^2}{2\Delta c}s^2 + \frac{(b_0 + \theta_0)^2}{2c_0}$$

or

$$B(s) = B_U - \frac{\Delta c(\alpha_A)^2 + c_A(\Delta a)^2}{c_A \Delta c}\frac{\tau^2}{2} = B_U - \frac{1}{2\gamma}s^2, \tag{B.1}$$

where B_U are counterfactual benefits (Equation (2.18)), i.e., benefits in the absence of regulation. Next, we insert the cut-offs into Equations (2.12) and (2.13), so we have total emissions

$$e(s) = a_A\left(\frac{b_A+\theta_A}{c_A}\right) - \Delta a\frac{\Delta b+\Delta\theta}{\Delta c} + a_0\frac{b_0+\theta_0}{c_0} - \frac{\alpha_A^2}{c_A}s - \frac{\Delta a^2}{\Delta c}s$$

or, after applying the definition of counterfactual emissions (Equation (2.19)), we have total emissions

$$e(s) = U - \frac{\Delta c\alpha_A^2 + c_A\Delta a^2}{c_A\Delta c}\tau = U - \frac{1}{\gamma}s. \tag{B.2}$$

Thus, we may write that

$$W(s) = B(s) - D(e(s)) = B_U - \frac{1}{2\gamma}s^2 - \frac{d}{2}\left(U - \frac{1}{\gamma}s\right)^2. \tag{B.3}$$

We further decompose counterfactual emissions into deterministic and stochastic parts. Then, we have

$$U = \overline{U} + \Phi, \tag{B.4}$$

where $\overline{U} = EU$ and $\Phi = a_A\left(\frac{\theta_A}{c_A}\right) - \Delta a\frac{\Delta\theta}{\Delta c}$.

We denote the optimal tax by τ. It satisfies the first order condition

$$E\left[\frac{dB}{d\tau} - \frac{dD}{de}\frac{de}{d\tau}\right] = 0$$

or, by Equation (B.3), the condition

$$\tau = \frac{\gamma d}{\gamma + d}\overline{U}. \tag{B.5}$$

Specifically, if we insert the optimal tax in Equation (B.5) into welfare, we have

$$
\begin{aligned}
W_\tau &= B_U - \frac{1}{2\gamma}\left(\frac{\gamma d}{\gamma+d}\overline{U}\right)^2 - \frac{d}{2}\left(U - \frac{1}{\gamma}\frac{\gamma d}{\gamma+d}\overline{U}\right)^2 \\
&= B_U - \frac{1}{2\gamma}\left(\frac{\gamma d}{\gamma+d}\overline{U}\right)^2 - \frac{d}{2}\left(\Phi + \left(1 - \frac{1}{\gamma}\frac{\gamma d}{\gamma+d}\right)\overline{U}\right)^2 \\
&= B_U - \frac{1}{2\gamma}\left(\frac{\gamma d}{\gamma+d}\overline{U}\right)^2 \\
&\quad - \frac{d}{2}\left(\left(\left(1 - \frac{1}{\gamma}\frac{\gamma d}{\gamma+d}\right)\overline{U}\right)^2 + \Phi^2 + 2\Phi\left(1 - \frac{1}{\gamma}\frac{\gamma d}{\gamma+d}\right)\overline{U}\right).
\end{aligned}
$$

As $E\Phi = 0$, we write that

$$
EW_\tau = EB_U - \left[\frac{1}{\gamma}\left(\frac{\gamma d}{\gamma+d}\right)^2 + d\left(1 - \frac{1}{\gamma}\frac{\gamma d}{\gamma+d}\right)^2\right]\frac{\overline{U}^2}{2} - \frac{d}{2}E\Phi^2.
$$

Furthermore, as

$$
\frac{1}{\gamma}\left(\frac{\gamma d}{\gamma+d}\right)^2 + d\left(1 - \frac{1}{\gamma}\frac{\gamma d}{\gamma+d}\right)^2 = \frac{\gamma d}{\gamma+d},
$$

it holds that

$$
EW_\tau = EB_U - \frac{\gamma d}{\gamma+d}\frac{\overline{U}^2}{2} - \frac{d}{2}E\Phi^2.
$$

By Equation (B.5), we may also write that

$$
EW_\tau = EB_U - \tau\frac{\overline{U}}{2} - \frac{d}{2}E\Phi^2. \tag{B.6}
$$

B.1.2 Imperfect Participation without Voluntary Participation

Consider next the imperfect participation. Following the steps in the previous section, we derive benefits as

$$
B(s_a) = \frac{(b_A + \theta_A)^2}{2c_A} - \frac{\alpha_A^2}{2c_A}s_a^2 + \frac{(\Delta b + \Delta\theta)^2}{2\Delta c} + \frac{(b_0 + \theta_0)^2}{2c_0} = B_U - \frac{s_a^2}{2\gamma_a} \tag{B.7}
$$

and emissions as

$$e(s_a) = a_A\left(\frac{b_A+\theta_A}{c_A}\right) - \Delta a \frac{\Delta b + \Delta\theta}{\Delta c} + a_0 \frac{b_0+\theta_0}{c_0} - \frac{\alpha_A^2}{c_A} s_a = U - \frac{1}{\gamma_a} s_a \tag{B.8}$$

in terms of the unit price s_a. We define the slope of the abatement cost function γ_a in Equation (2.24). Benefits and damages together will yield (a counterpart of Equation (B.3))

$$W_{\tau_a} = B_U - \frac{1}{2\gamma_a} s_a^2 - \frac{d}{2}\left(\overline{U} + \Phi - \frac{1}{\gamma_a} s_a\right)^2,$$

or, by using the optimal tax ($s_a = \tau_a$) in Equation (2.26), will yield

$$W_{\tau_a} = B_U - \frac{1}{2\gamma_a}\left(\frac{\gamma_a d}{\gamma_a + d}\right)^2 - \frac{d}{2}\left(\overline{U} + \Phi - \frac{1}{\gamma_a}\frac{\gamma_a d}{\gamma_a + d}\right)^2.$$

After taking the expected value, we may write that

$$EW_{\tau_a} = EB_U - \left[\frac{1}{\gamma_a}\left(\frac{\gamma_a d}{\gamma_a + d}\right)^2 + d\left(1 - \frac{1}{\gamma_a}\frac{\gamma_a d}{\gamma_a + d}\right)^2\right]\frac{E\overline{U}^2}{2} - \frac{d}{2}E\Phi^2$$

or, as the term inside the parenthesis equals $\frac{\tau_a}{E\overline{U}}$, write that

$$EW_{\tau_a} = EB_U - \tau_a \frac{\overline{U}}{2} - \frac{d}{2}E\Phi^2. \tag{B.9}$$

B.1.3 Voluntary Participation

As a third alternative, we consider expected social welfare under voluntary participation. Insert the cut-offs from Equations (2.39) into the definition of aggregate benefits (Equation (2.16)). We have

$$B(s_m) = \frac{(b_A+\theta_A)^2}{2c_A} - \frac{\alpha_A^2}{2c_A} s_m^2 + \frac{(\Delta b + \Delta\theta)^2}{2\Delta c} + \frac{(b_0+\theta_0)^2}{2c_0} - \frac{\varphi_m^2}{2\Delta c} s_m^2,$$

or, by Equations (2.18) and (2.43), we have

$$B(s_m) = B_U - \frac{\Delta c(\alpha_A)^2 + c_A\varphi_m^2}{c_A \Delta c}\frac{s_m^2}{2} = B_U - \frac{1}{2\gamma_m^l} s_m^2. \tag{B.10}$$

We calculate that

$$
\begin{aligned}
e_m &= \int_0^{\eta_A} a_A d\lambda + \int_0^{\lambda_{1m}} a_1 d\lambda + \int_{\lambda_{1m}}^{\lambda_0^0} a_0 d\lambda \\
&= a_A \left(\frac{b_A + \theta_A - s_m \alpha_A}{c_A} \right) - \Delta a \frac{\Delta b + \Delta \theta + s_m \varphi_m}{\Delta c} + a_0 \frac{b_0 + \theta_0}{c_0} \\
&= U - \frac{\Delta c \alpha_A^2 + c_A \Delta a \varphi_m}{c_A \Delta c} s_m = U - \frac{1}{\gamma_m^L} s_m .
\end{aligned}
\tag{B.11}
$$

Then, it holds that

$$
W(s_m) = B(s_m) - D(e(s_m)) = B_U - \frac{1}{2\gamma_m^l} s_m^2 - \frac{d}{2} \left(U - \frac{1}{\gamma_m^L} s_m \right)^2 . \tag{B.12}
$$

Next, we calculate the optimal tax rate. Denote the optimal rate by τ_m. First order condition states that

$$
\frac{dE\left[B(\tau_m) - D(e(\tau_m))\right]}{d\tau_m} = E\left[-\frac{1}{\gamma_m^l} \tau_m + \frac{d}{\gamma_m^L} \left(U - \frac{1}{\gamma_m^L} \tau_m \right) \right] = 0.
$$

By construction, B_U and U are independent of τ_m. The first-order condition can also be written as

$$
\left[-\frac{1}{\gamma_m^l} \tau_m + \frac{d}{\gamma_m^L} \left(\overline{U} - \frac{1}{\gamma_m^L} \tau_m \right) \right] = 0,
$$

where $\overline{U}$ denotes the expected counterfactual emissions. After arrangements, it holds that

$$
\tau_m = \left(\frac{d\gamma_m^l \gamma_m^L}{(\gamma_m^L)^2 + d\gamma_m^l} \right) \overline{U}. \tag{B.13}
$$

Furthermore, if we incorporate τ_m into the social welfare in Equation (B.12), we may write the welfare as

$$
\begin{aligned}
W_{\tau_m} &= B(\tau_m) - D(e(\tau_m)) \\
&= B_U - \frac{1}{2\gamma_m^l} \left(\frac{d\gamma_m^l \gamma_m^L}{(\gamma_m^L)^2 + d\gamma_m^l} \overline{U} \right)^2 - \frac{d}{2} \left(U - \frac{1}{\gamma_m^L} \left(\frac{d\gamma_m^l \gamma_m^L}{(\gamma_m^L)^2 + d\gamma_m^l} \overline{U} \right)^2 \right)^2 .
\end{aligned}
$$

We use Equation (B.4) and take the expected value. It then holds that

$$
\begin{aligned}
EW_{\tau_m} &= E\left[B(\tau_m) - D(e(\tau_m))\right] \\
&= EB_U - \frac{1}{2\gamma_m^l}\left(\frac{d\gamma_m^l \gamma_m^L}{(\gamma_m^L)^2 + d\gamma_m^l}\right)^2 \overline{U}^2 - \frac{d}{2}E\Phi^2 - \frac{d}{2}\left(1 - \frac{d\gamma_m^l}{(\gamma_m^L)^2 + d\gamma_m^l}\right)\overline{U}^2 \\
&= EB_U - \frac{d}{2}E\Phi^2 \\
&- \left(\frac{1}{\gamma_m^l}(d\gamma_m^l\gamma_m^L)^2 + d\left((\gamma_m^L)^2 + d\gamma_m^l - d\gamma_m^l\right)^2\right)\frac{\overline{U}^2}{2\left((\gamma_m^L)^2 + d\gamma_m^l\right)^2}.
\end{aligned}
$$

After a few lines of manipulations, it holds that

$$
EW_{\tau_m} = EB_U - \frac{d}{2}E\Phi^2 - d\,(\gamma_m^L)^2\,\frac{\overline{U}^2}{2\left((\gamma_m^L)^2 + d\gamma_m^l\right)}
$$

or, by the second-best tax rate in Equation (B.13), it also holds that

$$
EW_{\tau_m} \equiv E\left[B_{\tau_m} - D_{\tau_m}\right] = EB_U - \frac{d}{2}E\Phi^2 - \frac{\gamma_m^L}{\gamma_m^l}\tau_m\frac{\overline{U}}{2}. \tag{B.14}
$$

B.2 Characteristics of Optimal Policies

In this section, we prove claims that we make in the main text. More precisely, we have three distinct topics in three separate parts. To some extent, every topic compares implementations between different regulatory regimes.

B.2.1 Part I

In Section 2.2.3, we claim that $\tau_a > \tau$ and $E[e(\tau_a)] > E[e(\tau)]$. To prove these, we first write the condition in Equation (B.5) as

$$
\tau = \frac{\gamma d}{\gamma + d}EU,
$$

where U is the counterfactual emissions defined in Equation (2.19) and E is the expectation operator. Note also that

$$\gamma_a = n\gamma,$$

where

$$n = 1 + ua^2 > 1.$$

Then, using the tax rate τ_a from Equation (2.26) in the main text, we have a difference

$$\tau_a - \tau = \frac{n\gamma d}{n\gamma + d} EU - \frac{\gamma d}{\gamma + d} EU = \frac{\gamma d^2}{(\gamma + d)(n\gamma + d)}(n-1)EU > 0. \quad \text{(B.15)}$$

Furthermore, by Equations (B.2) and (B.8) above, we know that

$$e(\tau_a) = U - \frac{\tau_a}{\gamma_a} \text{ and } e(\tau) = U - \frac{\tau}{\gamma}, \quad \text{(B.16)}$$

so we have

$$\begin{aligned} E[e(\tau_a)] - E[e(\tau)] &= E\left[U - \frac{\tau_a}{n\gamma} - (U - \frac{\tau}{\gamma})\right] \quad \text{(B.17)} \\ &= E\left[-\frac{1}{n\gamma}\left(\frac{n\gamma d}{n\gamma + d}\right)EU + \frac{1}{\gamma}\left(\frac{\gamma d}{\gamma + d}\right)EU\right] \\ &= \gamma d\left(\frac{n-1}{(\gamma + d)(n\gamma + d)}\right)EU > 0. \end{aligned}$$

B.2.2 Part II

We show next that the regulator prefers perfect to imperfect participation (a claim made in Section 2.2.3). By Equations (B.6) and (B.9), we may conclude that

$$EW_{\tau_a} - EW_{\tau} = (\tau - \tau_a)\frac{EU}{2}.$$

We calculated above (Equation (B.15)) that $\tau - \tau_a < 0$. Thus, $EW_{\tau} > EW_{\tau_a}$.

B.2.3 Part III

In this final part, we provide a full description of optimal policies under various regulatory regimes. Specifically, we study various optimal prices and quantities that the optimal policies will induce.

We define in the main text that

$$k_m \equiv \frac{\varphi_m}{\alpha_A} \geq 0$$

and

$$a - k_m = \frac{\Delta\alpha - \varphi_m}{\alpha_A},$$

where $\varphi_1 = l_1 - \alpha_1 \geq 0$, $\varphi_2 \equiv \Delta\alpha - \Delta l \geq 0$ and $m = 1, 2.$ We calculate in Equation (B.11) that the emissions under voluntary participation are

$$e_m = U - \frac{1}{\gamma_m^L}\tau_m$$

and in Equation (B.13) that the optimal tax rate is

$$\tau_m = \frac{d\gamma_m^l \gamma_m^L}{(\gamma_m^L)^2 + d\gamma_m^l} EU. \tag{B.18}$$

We also calculate τ (Equation (B.5)), τ_a (Equation (2.26)), $E[e(\tau)]$ and $E[e(\tau_a)]$ (Equations (B.16)).

We calculate next differences between various policy variables. Instead of reporting the tedious but straightforward calculations, we write down the various results in Table B.1.

Table B.1 Various Differences under Tax Policies

$\tau_m - \tau = \gamma d \left(\frac{\gamma(F-\rho)+d(F-1)}{(\gamma+d)(\gamma\rho+d)} \right) EU$
$\tau_m - \tau_a = \gamma d \left(\frac{n\gamma(F-\rho)+d(F-n)}{(n\gamma+d)(\gamma\rho+d)} \right) EU$
$E[e(\tau_m)] - E[e(\tau)] = \gamma d \left(\frac{\rho-1}{(\gamma+d)(\gamma\rho+d)} \right) EU$
$E[e(\tau_m)] - E[e(\tau_a)] = \gamma d \left(\frac{\rho-n}{(\gamma\rho+d)(n\gamma+d)} \right) EU$

In Table B.1, we have

$$F = \frac{\gamma_m^L}{\gamma} = \frac{1+ua^2}{1+uak_m} \tag{B.19}$$

and

$$\rho = \frac{(\gamma_m^L)^2}{\gamma\gamma_m^l} = \frac{\left(1+uk_m^2\right)\left(1+ua^2\right)}{(1+uak_m)^2} = 1 + \frac{u\,(k_m-a)^2}{(1+uak_m)^2} \tag{B.20}$$

so that

$$\frac{F}{\rho} = \frac{1+uak_m}{1+uk_m^2}.$$

Furthermore, as

$$n = 1+ua^2,$$

(see Equation (2.28) in the main text), then

$$\frac{n}{F} = 1+uak_m$$

and

$$\frac{n}{\rho} = \frac{(1+uak_m)^2}{1+uk_m^2}. \tag{B.21}$$

Note that $ua > 0$ and $\rho \equiv \rho_m$.

Overall, we have either $k_1 = \frac{l_1-\alpha_1}{\alpha_A} \geq 0$ or $k_2 = \frac{(l_1-\alpha_1)-(l_0-\alpha_0)}{\alpha_A} \geq 0$. Furthermore, we have either $a-k_1 = \frac{\alpha_0-l_1}{\alpha_A}$ or $a-k_2 \equiv \frac{\Delta l}{\alpha_A}$. The zero voluntary participation means $k_m = 0$ and $k_m > 0$ means strictly positive voluntary participation among firms in green technology. Let $k_m > 0$. Then, by the efficiency conditions calculated in the main text (Equations (2.37) and (2.38)), the condition $a-k_m = 0$ refers to an efficient implementation and the condition $a-k_m \neq 0$ to an inefficient one.

Now, based on Table B.1, we may write:

i. $\tau_m - \tau = 0$ under efficiency. As long as $a = k_m > 0$, we have $F = \rho = 1$. However, if $a \neq k_m$, then the sign of $\tau_m - \tau$ is ambiguous.

ii. If no firm participates, then $\tau_m = \tau_a$. We have $k_m = 0$, so $F = \rho = n$. However, if $k_m > 0$, then the sign of $\tau_m - \tau_a$ is ambiguous.

iii. $E[e(\tau_m)] = E[e(\tau)]$ under efficiency. This follows as the condition $a = k_m > 0$ implies that $\rho = 1$. If $a \neq k_m$, instead, then $\rho > 1$ and $E[e(\tau_m)] > E[e(\tau)]$.

iv. If $k_m = 0$, then $\rho = n$ and $E[e(\tau_m)] = E[e(\tau_a)]$. However, $E[e(\tau_m)] <$

$E[e(\tau_a)]$ only if $n > \rho$.

B.3 Combined Cost-Scale-Volume Effect

This section derives a magnitude for the total effect that the voluntary participation has on instrument choice. This means that we combine the influences of cost, scale and volume effects to see whether voluntary participation favors prices or quantities. In Part I, we ignore the scale effect, while in Part II, the scale effect is wholly included into the total effect.

In our derivations, we will employ the scale effect

$$n = 1 + ua^2$$

(see Equation (2.28) in the main text) and the cost effect

$$\rho = \frac{\left(1+uk^2\right)\left(1+ua^2\right)}{(1+uak)^2}$$

(see Equation (A.9) in appendix A.9). Furthermore, the volume effect is

$$\Theta \equiv \Theta(\tau, p) = 2q - 1,$$

where

$$q = \frac{1+uk^2}{1+uka}\frac{1+u^2ka}{1+u^2k^2}$$

(see Equations (2.68) and (2.70) in the main text).

B.3.1 Part I

It holds that

$$
\begin{aligned}
&\Theta(\tau,p)-\rho=2q-1-\rho=\\
&=2\left(\frac{1+uk^2}{1+uka}\frac{1+u^2ka}{1+u^2k^2}\right)-1-\frac{\left(1+uk^2\right)\left(1+ua^2\right)}{(1+uak)^2}\\
&=\frac{1+uk^2}{1+uka}\left(2\frac{1+u^2ka}{1+u^2k^2}-\frac{1+uka}{1+uk^2}+-\frac{1+ua^2}{1+uka}\right).
\end{aligned}
$$

By straightforward calculations, it also holds that

$$
\begin{aligned}
&\Theta(\tau,p)-\rho\\
&=\frac{1+uk^2}{1+uka}\left[\left(\frac{1+u^2ka}{1+u^2k^2}-\frac{1+uka}{1+uk^2}\right)+\left(\frac{1+u^2ka}{1+u^2k^2}-\frac{1+ua^2}{1+uka}\right)\right]\\
&=\frac{1+uk^2}{1+uka}\left(\frac{\left(1+u^2ka\right)\left(1+uk^2\right)-\left(1+u^2k^2\right)(1+uka)}{(1+u^2k^2)(1+uk^2)}\right)\\
&+\frac{1+uk^2}{1+uka}\left(\frac{\left(1+u^2ka\right)(1+uka)-\left(1+u^2k^2\right)\left(1+ua^2\right)}{(1+u^2k^2)(1+uka)}\right)\\
&=uk\frac{1+uk^2}{1+uka}\left[\left(\frac{ua+k-uk-a}{(1+u^2k^2)(1+uk^2)}\right)+\left(\frac{ua+a-uk-\frac{a^2}{k}}{(1+u^2k^2)(1+uka)}\right)\right]\\
&=uk\frac{1+uk^2}{1+uka}(a-k)\left[\left(\frac{u-1}{(1+u^2k^2)(1+uk^2)}\right)+\left(\frac{u+\left(1-\frac{a}{k}\right)\frac{a}{a-k}}{(1+u^2k^2)(1+uka)}\right)\right]\\
&=uk\frac{1+uk^2}{1+uka}\frac{(a-k)}{(1+u^2k^2)}\left[\left(\frac{u-1}{(1+uk^2)}\right)+\left(\frac{u-\frac{a}{k}}{(1+uka)}\right)\right].
\end{aligned}
$$

B.3.2 Part II

We have

$$
\Theta-\frac{\rho}{n}=2q-1-\frac{\rho}{n},
$$

or in terms of u, k, and a,

$$\Theta - \frac{\rho}{n} = 2\left(\frac{1+uk^2}{1+uka}\frac{1+u^2ka}{1+u^2k^2}\right) - 1 - \frac{\left(1+uk^2\right)}{(1+uak)^2}$$
$$= \frac{1+uk^2}{1+uka}\left(2\frac{1+u^2ka}{1+u^2k^2} - \frac{1+uka}{1+uk^2} - \frac{1}{1+uka}\right).$$

Then, we calculate that

$$\begin{aligned}
&\Theta - \frac{\rho}{n} \\
&= \frac{1+uk^2}{1+uka}\left[\left(\frac{1+u^2ka}{1+u^2k^2} - \frac{1+uka}{1+uk^2}\right) + \left(\frac{1+u^2ka}{1+u^2k^2} - \frac{1}{1+uka}\right)\right] \\
&= uk\frac{1+uk^2}{1+uka}\left[\left(\frac{ua+k-uk-a}{(1+u^2k^2)(1+uk^2)}\right) + \left(\frac{ua+a+u^2k^2a^2-uk}{(1+u^2k^2)(1+uka)}\right)\right] \\
&= uk\frac{1+uk^2}{1+uka}\left[\left(\frac{u(a-k)-(a-k)}{(1+u^2k^2)(1+uk^2)}\right) + \left(\frac{u(a-k)+a\left(1+u^2k^2a\right)}{(1+u^2k^2)(1+uka)}\right)\right] \\
&= uk\frac{1+uk^2}{1+uka}\left(\frac{u(a-k)-(a-k)}{(1+u^2k^2)(1+uk^2)}\right) \\
&+ uk\frac{1+uk^2}{1+uka}\left(\frac{u(a-k)+a+k-k+u^2k^2a^2}{(1+u^2k^2)(1+uka)}\right) \\
&= uk\frac{(a-k)}{(1+u^2k^2)}\frac{1+uk^2}{1+uka}\left[\left(\frac{u-1}{(1+uk^2)}\right) + \left(\frac{u+1+\frac{k\left(1+ku^2a^2\right)}{(a-k)}}{(1+uka)}\right)\right].
\end{aligned}$$

B.4 A General Representation

Section 2.3.7.1 in the main text presents a general formula for instrument choice. It applies in both prices versus quantities and in quantities versus quantities comparisons. We will derive the formula here.

Our writing proceeds in parts. Within the different phases, we will apply definitions:

$$q(s) = \frac{R_0(s)\left(\frac{\alpha_A}{c_A}\right)^2 + R_1(s)\left(\frac{\Delta\alpha}{\Delta c}\right)^2}{(R_0(s))^2\left(\frac{\alpha_A}{c_A}\right)^2 + (R_1(s))^2\left(\frac{\Delta\alpha}{\Delta c}\right)^2} \tag{B.22}$$

and

$$v = v(I,J) = \frac{Var(I)}{Var(J)}. \tag{B.23}$$

Calculate then

$$\frac{Var(e(I) - Var(e(J)}{Var(J) - Var(I)} = -\frac{[Var(e(J) - Var(e(I)]}{Var(J) - Var(I)},$$

where (by Equations (2.81) and (2.82) in the main text)

$$Var(I) = \left(\gamma^L\right)^2 \sigma^2 \left[\left(R_0(I)\frac{\alpha_A}{c_A}\right)^2 + \left(R_1(I)\frac{\Delta\alpha}{\Delta c}\right)^2\right]$$

and

$$Var(e(I)) = \sigma^2 \left[\left((1 - R_0(I))\frac{\alpha_A}{c_A}\right)^2 + \left((1 - R_1(I))\frac{\Delta\alpha}{\Delta c}\right)^2\right].$$

We manipulate first the numerator. We write it as

$$\begin{aligned} Var(e(J) - Var(e(I) &= 2\left(R_0(J)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2\right) \\ &- 2\left(R_0(I)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(I)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2\right) - \frac{Var(J) - Var(I)}{\left(\gamma_m^L\right)^2} \end{aligned}$$

or, after taking $R_0(J)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2$ as a common factor, as

$$\begin{aligned} Var(e(J) - Var(e(I) &= 2\left(R_0(J)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2\right) \\ &\left(1 - \frac{R_0(I)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(I)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2}{R_0(J)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2}\right) - \frac{Var(J) - Var(I)}{\left(\gamma_m^L\right)^2}. \end{aligned}$$

By application of Equation (B.22), it holds that

$$\frac{Var(J)}{Var(J)}\left(R_0(J)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2\right) = \frac{Var(J)}{\left(\gamma_m^L\right)^2}(q(J)).$$

Furthermore, by applications of Equations (B.22) and (B.23), it also holds that

$$\frac{R_0(I)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(I)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2}{R_0(J)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2}$$

$$= \frac{\left(R_0(I)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(I)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2\right)\frac{(R_0(I))^2\left(\frac{\alpha_A}{c_A}\theta_A\right)^2+(R_1(I))^2\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2}{(R_0(I))^2\left(\frac{\alpha_A}{c_A}\theta_A\right)^2+(R_1(I))^2\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2}}{\left(R_0(J)\left(\frac{\alpha_A}{c_A}\theta_A\right)^2 + R_1(J)\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2\right)\frac{(R_0(J))^2\left(\frac{\alpha_A}{c_A}\theta_A\right)^2+(R_1(J))^2\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2}{(R_0(J))^2\left(\frac{\alpha_A}{c_A}\theta_A\right)^2+(R_1(J))^2\left(\frac{\Delta\alpha}{\Delta c}\Delta\theta\right)^2}} = v\frac{q(I)}{q(J)}.$$

Now, we may write that

$$Var(e(J) - Var(e(I) = 2\frac{Var(J)}{\left(\gamma_m^L\right)^2}(q(J))\left(1 - v\frac{q(I)}{q(J)}\right) - \frac{Var(J) - Var(I)}{\left(\gamma_m^L\right)^2}$$

$$= \frac{Var(J)}{\left(\gamma_m^L\right)^2}\left(2(q(J))\left(1 - v\frac{q(I)}{q(J)}\right) - (1-v)\right).$$

As

$$Var(J) - Var(I) = Var(J)(1-v),$$

we may finally write that

$$\frac{Var(e(I) - Var(e(J)}{Var(J) - Var(I)} = \frac{\frac{Var(J)}{\left(\gamma_m^L\right)^2}\left(2(q(J))\left(1 - v\frac{q(I)}{q(J)}\right) - (1-v)\right)}{Var(J)(1-v)}$$

$$= \frac{1}{\left(\gamma_m^L\right)^2}\left(2(q(J))\frac{\left(1 - v\frac{q(I)}{q(J)}\right)}{(1-v)} - 1\right).$$

B.5 The Relation between Quantity and Price Variances

We want to link price and quantity variations to each other. By Equation (2.82), it holds that

$$Var(e(s)) = \sigma^2\left[\left((1 - R_0(s))\frac{\alpha_A}{c_A}\right)^2 + \left((1 - R_1(s))\frac{\Delta\alpha}{\Delta c}\right)^2\right].$$

Expand it to get

$$
\begin{aligned}
Var(e(s)) = {} & \sigma^2(1-2R_0(I)+R_0(I))^2)\left(\frac{\alpha_A}{c_A}\right)^2 \\
& +\sigma^2\left(1-2R_1(I)+R_1(I))^2\right)\left(\frac{\Delta\alpha}{\Delta c}\right)^2.
\end{aligned}
$$

After arranging,

$$
\begin{aligned}
Var(e(s)) = {} & \sigma^2\left(\left(\frac{\alpha_A}{c_A}\right)^2+\left(\frac{\Delta\alpha}{\Delta c}\right)^2\right) \\
& +\sigma^2 R_0(s))^2\left(\frac{\alpha_A}{c_A}\right)^2\left(1-2\frac{\left(R_0(s))\left(\frac{\alpha_A}{c_A}\right)^2+R_1(s)\left(\frac{\Delta\alpha}{\Delta c}\right)^2\right)}{R_0(s))^2\left(\frac{\alpha_A}{c_A}\right)^2+R_1(s))^2\left(\frac{\Delta\alpha}{\Delta c}\right)^2}\right) \\
& +\sigma^2 R_1(s))^2\left(\frac{\Delta\alpha}{\Delta c}\right)^2\left(1-2\frac{\left(R_0(s))\left(\frac{\alpha_A}{c_A}\right)^2+R_1(s)\left(\frac{\Delta\alpha}{\Delta c}\right)^2\right)}{R_0(s))^2\left(\frac{\alpha_A}{c_A}\right)^2+R_1(s))^2\left(\frac{\Delta\alpha}{\Delta c}\right)^2}\right).
\end{aligned}
$$

We introduce the counterfactual emissions U in Equation (2.19), $q(s)$ in Equation (2.76), and $Var(s)$ in Equation (2.81), so we may further write that

$$
Var(e(s)) = Var(U) - Var(s)\frac{2q(s)-1}{(\gamma^L)^2}.
$$

C MULTIPLE EXTERNALITIES

C.1 Generalized Linear Externality Effect

We present here a generalized linear externality effect. Let

$$F_i^k = \phi_{ij}^{kk} \lambda_j^k + \phi_{ii}^{kk} \lambda_i^k + \phi_{ij}^{kl} \lambda_j^l + \phi_{ii}^{kl} \lambda_i^l, \tag{C.1}$$

where $i, j = 0, 1$, $i \neq j$, $k, l = g, r$ and $k \neq l$. For example, λ_j^l is the number of firms in sector l that use technology j, while the co-efficient ϕ_{ij}^{kl} indicates the influence that the use of technology j in sector l has on the profitability of technology i in sector k. As Equation (C.1) shows, the externalities can flow between and within the technologies and sectors. Within a sector, the externality can prosper within the same technology ($\phi_{ii}^{kk} \lambda_i^k$) or between the technologies ($\phi_{ij}^{kk} \lambda_j^k$). The flow between industries may occur among the technologies ($\phi_{ii}^{kl} \lambda_i^l$) or between them ($\phi_{ij}^{kl} \lambda_j^l$).

We substantially restrict the externality flows in the model of the main text. First, we assume no externalities between the technologies. In other words, we set $\phi_{ij}^{kk} = \phi_{ij}^{kl} = 0$. Second, based on close inspection of the benefits, the effects $\phi_{ii}^{kk} \lambda_i^k$ and $\phi_i^k \lambda_i^k$ both depend on unit λ_i^k, so we effectively have a combined factor equal to $\left(c_i^k + \phi_{ii}^{kk}\right) \lambda_i^k$ in the benefits. In this particular case, we may say that the sector is able to internalize the externality. Thus, without loss of generality, we may set $\phi_{ii}^{kk} = 0$, as well. Third, we assume that $\phi_{00}^{kl} = 0$. That is, within the use of technology zero, there exists no externalities. Finally, we restrict the direction of the externality flow. We assume that sector g is an externality generator while sector r is an externality recipient, so $\phi_{11}^{gr} = 0$ while $\phi_{11}^{rg} \neq 0$. In summary, given the diversity of plausible externality flows, we will concentrate on only one of them. We denote $\phi = \phi_{11}^{rg}$.

C.2 Augmented Abatement Cost Function

In reducing the total emissions, we refer to abatement costs function as the minimum cost function. In the present context, the abatement cost function is not standard mainly because of the positive externality in production. We provide here the proper marginal abatement cost function, which turns out to be linear in emissions.

We write the efficient units (Equations (3.18) and (3.19)) as

$$\begin{aligned}\lambda_e^g &= \frac{\Delta c^r(\Delta b^g - \Delta\theta^g) + \phi(\Delta b^r - \Delta\theta^r)}{\Delta c^g \Delta c^r - \phi^2} + \mu\frac{\Delta\alpha^r\phi + \Delta\alpha^g\Delta c^r}{\Delta c^g \Delta c^r - \phi^2} \\ &= A^g(\theta) + C^g\mu\end{aligned} \tag{C.2}$$

and

$$\begin{aligned}\lambda_e^r &= \frac{\phi(\Delta b^g - \Delta\theta^g) + \Delta c^g(\Delta b^r - \Delta\theta^r)}{\Delta c^g \Delta c^r - \phi^2} + \mu\frac{\Delta\alpha^r\Delta c^g + \Delta\alpha^g\phi}{\Delta c^g \Delta c^r - \phi^2} \\ &= A^r(\theta) + C^r\mu.\end{aligned} \tag{C.3}$$

Next, we incorporate these units into emission constraint (Equation (3.16)). This will yield

$$\begin{aligned}e &= \lambda_0^{g+}\alpha_0^g - \Delta\alpha^g\lambda_1^g + \lambda_0^{r+}\alpha_0^r - \Delta\alpha^r\lambda_1^r = e^0 - \Delta\alpha^g\lambda_1^g - \Delta\alpha^r\lambda_1^r \\ &= e^0 - \Delta\alpha^g(A^g(\theta) + C^g\mu) - \Delta\alpha^r(A^r(\theta) + C^r\mu),\end{aligned}$$

or, after arrangements, it will yield

$$e = e^0 - \Delta\alpha^g A^g(\theta) - \Delta\alpha^r A^r(\theta) + (\Delta\alpha^g C^g + \Delta\alpha^r C^r)\mu.$$

By using definitions in Equations (C.2) and (C.3), we calculate that

$$\begin{aligned}\Delta\alpha^g C^g + \Delta\alpha^r C^r &= \Delta\alpha^g\left(\frac{\Delta\alpha^r\phi + \Delta\alpha^g\Delta c^r}{\Delta c^g\Delta c^r - \phi^2}\right) + \Delta\alpha^r\left(\frac{\Delta\alpha^r\Delta c^g + \Delta\alpha^g\phi}{\Delta c^g\Delta c^r - \phi^2}\right) \\ &= \frac{(\Delta\alpha^g)^2\Delta c^r + 2\Delta\alpha^g\Delta\alpha_1^r\phi + (\Delta\alpha^r)^2\Delta c^g}{\Delta c^g\Delta c^r - \phi^2} = \frac{1}{c}\end{aligned}$$

and

$$
\begin{aligned}
&-(\Delta\alpha^g A^g(\theta)+\Delta\alpha^r A^r(\theta)) = -\Delta\alpha^g\left(\frac{\Delta c^r(\Delta b^g-\Delta\theta^g)+\phi(\Delta b^r-\Delta\theta^r)}{\Delta c^g\Delta c^r-\phi^2}\right)\\
&-\Delta\alpha^r\left(\frac{\phi(\Delta b^g-\Delta\theta^g)+\Delta c^g(\Delta b^r-\Delta\theta^r)}{\Delta c^g\Delta c^r-\phi^2}\right)\\
&= e^{00}+\Delta\alpha^g\left(\frac{\Delta c^r\Delta\theta^g+\phi\Delta\theta^r}{\Delta c^g\Delta c^r-\phi^2}\right)+\Delta\alpha^r\left(\frac{\phi\Delta\theta^g+\Delta c^g\Delta\theta^r}{\Delta c^g\Delta c^r-\phi^2}\right)\\
&= e^{00}+\left(\frac{\Delta\alpha^g\Delta c^r+\Delta\alpha^r\phi}{\Delta c^g\Delta c^r-\phi^2}\right)\Delta\theta^g+\left(\frac{\Delta\alpha^r\Delta c^g+\Delta\alpha^g\phi}{\Delta c^g\Delta c^r-\phi^2}\right)\Delta\theta^r,
\end{aligned}
$$

where c is defined in Equation (3.20). Using these (and letting $A=e^0+e^{00}$), we may write that

$$
e(\mu;\Delta\theta^g,\Delta\theta^r)=A+\left(\frac{\Delta\alpha^g\Delta c^r+\Delta\alpha^r\phi}{\Delta c^g\Delta c^r-\phi^2}\right)\Delta\theta^g+\left(\frac{\Delta\alpha^r\Delta c^g+\Delta\alpha^g\phi}{\Delta c^g\Delta c^r-\phi^2}\right)\Delta\theta^r-\frac{\mu}{c}
$$

and

$$
\begin{aligned}
\mu(e;\Delta\theta^g,\Delta\theta^r) &= cA-Ce \\
&\quad +c\left[\left(\frac{\Delta\alpha^g\Delta c^r+\Delta\alpha^r\phi}{\Delta c^g\Delta c^r-\phi^2}\right)\Delta\theta^g+\left(\frac{\Delta\alpha^r\Delta c^g+\Delta\alpha^g\phi}{\Delta c^g\Delta c^r-\phi^2}\right)\Delta\theta^r\right].
\end{aligned}
\tag{C.4}
$$

In particular, by setting $\Delta\theta^g=\Delta\theta^r=0$, we have

$$
e(\mu)=A-\frac{\mu}{c}
$$

and

$$
\mu(e)=cA-ce.
$$

C.3 Efficient Subsidy Rule

In this section, we discuss the implementation of the efficient policy (Section 3.2.4). First order conditions for an efficient solution are

$$
\begin{cases}
\Delta b^g-\Delta c^g\lambda_e^g+\phi\lambda_e^r+\mu\Delta\alpha^g=0\\
\Delta b^r-\Delta c^r\lambda_e^r+\phi\lambda_e^g+\mu\Delta\alpha^r=0
\end{cases},
$$

while the market choices λ_1^g and λ_1^r satisfy

$$\begin{cases} \Delta b^g - \Delta c^g \lambda_1^g + s\Delta\alpha^g + \Delta S^g = 0 \\ \Delta b^r - \Delta c^r \lambda_1^r + \phi\lambda_1^g + s\Delta\alpha^r + \Delta S^r = 0 \end{cases}$$

or, equivalently, satisfy

$$\begin{cases} \Delta b^g - \Delta c^g \lambda_1^g + \phi\lambda_1^r - \phi\lambda_1^r + s\Delta\alpha^g + \Delta S^g = 0 \\ \Delta b^r - \Delta c^r \lambda_1^{r*} + \phi\lambda_1^g + s\Delta\alpha^r + \Delta S^r = 0 \end{cases}$$

(see Equations (3.17) and (3.21)). The efficient implementation requires that $\lambda_e^g = \lambda_1^g$ and $\lambda_e^r = \lambda_1^r$. Therefore, by the conditions above, it must hold that

$$\begin{cases} \mu\Delta\alpha^g = -\phi\lambda_1^r + s\Delta\alpha^g + \Delta S^g \\ \mu\Delta\alpha^r = s\Delta\alpha^r + \Delta S^r \end{cases} \tag{C.5}$$

or that

$$\frac{-\phi\lambda_1^r + s\Delta\alpha^g + \Delta S^g}{\Delta\alpha^g} = \frac{s\Delta\alpha^r + \Delta S^r}{\Delta\alpha^r}.$$

In specific, as for the relationship between the payment rules, we arrange the previous equation to get

$$\Delta S^g = \frac{\Delta\alpha^g}{\Delta\alpha^r}\Delta S^r + \phi\lambda_1^r.$$

Furthermore, by Equation (C.5), it holds between μ and s that

$$\mu = s + \frac{\Delta S^r}{\Delta\alpha^r}$$

in the efficient allocation.

C.4 Social Welfare Maximization

We will describe next the optimal societal policy (Section 3.3.1). The policy will maximize the difference between benefits and damages of emissions and is implemented by simultaneous use of emission price and sector specific thresholds.

C.4.1 Part I

We will solve the following problem:

$$\underset{\tau,\Delta S^g,\Delta S^r}{Max} EW = E\left[\int_0^{\lambda_1^g} B_1^g d\lambda^g + \int_{\lambda_1^g}^{\lambda_0^{g+}} B_0^g d\lambda^g + \int_0^{\lambda_1^r} B_1^r d\lambda^r + \int_{\lambda_1^r}^{\lambda_0^{r+}} B_0^r d\lambda^r\right]$$
$$-E\left[D(e(\lambda_1^g,\lambda_1^r))\right],$$

where

$$\lambda_1^g = \lambda_1^g(\tau,\Delta S^g) = \frac{\Delta b^g - \Delta\theta^g + \tau\Delta\alpha^g + \Delta S^g}{\Delta c^g} \tag{C.6}$$

and

$$\begin{aligned}\lambda_1^r = \lambda_1^r(\tau,\Delta S^g,\Delta S^r) &= \frac{\Delta b^r - \Delta\theta^r + \tau\Delta\alpha^r + \Delta S^r + \phi\lambda_1^g(\tau,\Delta S^g)}{\Delta c^r} \\ &= \frac{\Delta b^r - \Delta\theta^r}{\Delta c^r} + \frac{(\phi\Delta\alpha^g + \Delta c^g\Delta\alpha^r)}{\Delta c^g\Delta c^r}\tau + \frac{\Delta c^g\Delta S^r + \phi\Delta S^g}{\Delta c^g\Delta c^r}\end{aligned} \tag{C.7}$$

are the sector-specific responses calculated in Equations (3.5), (3.13), and (3.14), respectively. We may also write the benefits as

$$\begin{aligned}B(\lambda_1^g,\lambda_1^r) = &\int_0^{\lambda_1^g} \left(b_1^g + \theta_1^g - c_1^g\lambda^g\right) d\lambda^g + \int_{\lambda_1^g}^{\lambda_0^{g+}} \left(b_0^g + \theta_0^g - c_0^g\lambda^g\right) d\lambda^g \\ &+ \int_0^{\lambda_1^r} \left(b_1^r + \theta_1^r - c_1^r\lambda^r + \phi\lambda_1^g\right) d\lambda^r + \int_{\lambda_1^r}^{\lambda_0^{r+}} (b_0^r + \theta_0^r - c_0^r\lambda^r) d\lambda^r\end{aligned}$$

and the emissions as

$$\begin{aligned}e &= \int_0^{\lambda_1^g} \alpha_1^g d\lambda + \int_{\lambda_1^g}^{\lambda_0^{g+}} \alpha_0^g d\lambda + \int_0^{\lambda_1^r} \alpha_1^r d\lambda + \int_{\lambda_1^r}^{\lambda_0^{r+}} \alpha_0^r d\lambda \\ &= \lambda_0^{g+}\alpha_0^g - \Delta\alpha^g\lambda_1^g + \lambda_0^{r+}\alpha_0^r - \Delta\alpha^r\lambda_1^r.\end{aligned}$$

First order conditions are

$$\frac{dEW}{d\tau} = E\left[(B_1^g(\lambda_1^g) - B_0^g(\lambda_1^g) + \phi\lambda_1^r + \Delta\alpha^g D'(e))\frac{d\lambda_1^g}{d\tau}\right] \tag{C.8}$$
$$+ E\left[(B_1^r(\lambda_1^r) - B_0^r(\lambda_1^r) + \Delta\alpha^r D'(e))\frac{d\lambda_1^r}{d\tau}\right]$$
$$= E\left[(B_1^g(\lambda_1^g) - B_0^g(\lambda_1^g) + \phi\lambda_1^r + \Delta\alpha^g D'(e))\right]\frac{d\lambda_1^g}{d\tau}$$
$$+ E\left[(B_1^r(\lambda_1^r) - B_0^r(\lambda_1^r) + \Delta\alpha^r D'(e))\right]\frac{d\lambda_1^r}{d\tau}$$
$$= 0,$$

$$\frac{dEW}{d\Delta S^g} = E\left[(B_1^g(\lambda_1^g) - B_0^g(\lambda_1^g) + \phi\lambda_1^r + \Delta\alpha^g D'(e))\frac{d\lambda_1^g}{\Delta S^g}\right] \tag{C.9}$$
$$+ E\left[(B_1^r(\lambda_1^r) - B_0^r(\lambda_1^r) + \Delta\alpha^r D'(e))\frac{d\lambda_1^r}{\Delta S^g}\right]$$
$$= E\left[(B_1^g(\lambda_1^g) - B_0^g(\lambda_1^g) + \phi\lambda_1^r + \Delta\alpha^g D'(e))\right]\frac{d\lambda_1^g}{\Delta S^g}$$
$$+ E\left[(B_1^r(\lambda_1^r) - B_0^r(\lambda_1^r) + \Delta\alpha^r D'(e))\right]\frac{d\lambda_1^r}{\Delta S^g}$$
$$= 0,$$

and

$$\frac{dEW}{d\Delta S^r} = E\left[(B_1^r(\lambda_1^r) - B_0^r(\lambda_1^r) + \Delta\alpha^r D'(e))\frac{d\lambda_1^r}{d\Delta S^r}\right] \tag{C.10}$$
$$= E\left[(B_1^r(\lambda_1^r) - B_0^r(\lambda_1^r) + \Delta\alpha^r D'(e))\right]\frac{d\lambda_1^r}{d\Delta S^r} = 0.$$

Note that these conditions reduces to two conditions

$$E\left[(B_1^g(\lambda_1^g) - B_0^g(\lambda_1^g) + \phi\lambda_1^r + \Delta\alpha^g D'(e))\right] = 0 \tag{C.11}$$

and

$$E\left[(B_1^r(\lambda_1^r) - B_0^r(\lambda_1^r) + \Delta\alpha^r D'(e))\right] = 0. \tag{C.12}$$

Next, we calculate (by using responses in Equations (C.6) and (C.7)) that

$$
\begin{aligned}
&B_1^g(\lambda_1^g(\tau,\Delta S^g))-B_0^g(\lambda_1^g(\tau,\Delta S^g))\\
&=b_1^g+\theta_1^g-c_1^g\lambda_1^g(\tau,\Delta S^g)-\left(b_0^g+\theta_0^g-c_0^g\lambda_1^g(\tau,\Delta S^g)\right)\\
&=\Delta b^g-\Delta\theta^g-\Delta c^g\lambda_1^g(\tau,\Delta S^g)\\
&=\Delta b^g-\Delta\theta^g-\Delta c^g\left(\frac{\Delta b^g-\Delta\theta^g+\tau\Delta\alpha^g+\Delta S^g}{\Delta c^g}\right)\\
&=-(\tau\Delta\alpha^g+\Delta S^g)
\end{aligned}
$$

and

$$
\begin{aligned}
&B_1^r(\lambda_1^r(\tau,\Delta S^g,\Delta S^r))-B_0^r(\lambda_1^r(\tau,\Delta S^g,\Delta S^r))\\
&=b_1^r+\theta_1^r-c_1^r\lambda_1^r\lambda_1^r(\tau,\Delta S^g,\Delta S^r)+\phi\lambda_1^g(\tau,\Delta S^g)\\
&=\Delta b^r-\Delta\theta^r-\Delta c^g\lambda_1^r(\tau,\Delta S^g,\Delta S^r)+\phi\lambda_1^g(\tau,\Delta S^g)\\
&=\Delta b^r-\Delta\theta^r+\phi\lambda_1^g(\tau,\Delta S^g)\\
&-\Delta c^r\left(\frac{\Delta b^r-\Delta\theta^r+\tau\Delta\alpha^r+\Delta S^r+\phi\lambda_1^g(\tau,\Delta S^g)}{\Delta c^r}\right)\\
&=-(\tau\Delta\alpha^r+\Delta S^r).
\end{aligned}
$$

Then, condition in Equation (C.12) can be written as

$$E\left[(-(\tau\Delta\alpha^r+\Delta S^r)+\Delta\alpha^r D'(e))\right]=0$$

or as

$$\tau-ED'(e)=-\frac{\Delta S^r}{\Delta\alpha^r}.$$

Second, the condition in Equation (C.11) becomes

$$E\left[-(\tau\Delta\alpha^g+S^g)+\phi E\lambda_1^r+\Delta\alpha^g D'(e)\right]=0$$

so that

$$\tau-ED'(e)=\frac{-S^g+\phi E\lambda_1^r(\tau,\Delta\theta^r)}{\Delta\alpha^g}.$$

Specifically, it holds that

$$\Delta S^g = \frac{\Delta\alpha^g}{\Delta\alpha^r}\Delta S^r - \phi E\lambda_1^r(\theta)$$

at the optimum.

We make the same comment here as we did earlier in Chapter 1, appendix A.1. The tax rate is not a compulsory part in the implementation of the optimal policy. As long as conditions in Equations (C.9) and (C.10) are satisfied, then the condition in Equation (C.8) automatically holds. In other words, it must only hold that

$$\Delta S^r = \Delta\alpha^r D'(e)$$

and

$$\Delta S^g = \Delta\alpha^g D'(e) - \phi\lambda_1^r$$

at the optimum. In this solution, the payments reflect the type of the externality. The payments $\Delta\alpha^r D'(e)$ and $\Delta\alpha^g D'(e)$ are related to the negative externality while the payment $\phi\lambda_1^r$ is related to the positive externality. In the absence of positive externalities, only the parts $\Delta\alpha^r D'(e)$ and $\Delta\alpha^g D'(e)$ survive.

However, the agency has no reason to dismiss the market-based instruments from the outset. The reason is that the abandonment will automatically fix the payments. Later on, when we study the instrument choice, the fluctuating payments (provided by permit trading) may well yield higher expected welfare than the fixed payments.

C.4.2 Part II

We note that the conditions in Equations (C.11) and (C.12) can also be written as

$$\begin{cases} E\left[\Delta b^g - \Delta\theta^g - \Delta c^g\lambda_o^g + \phi\lambda^r + D'(e)\Delta\alpha^g\right] = 0 \\ E\left[\Delta b^r - \Delta\theta^r - \Delta c^r\lambda_o^r + \phi\lambda^g + D'(e)\Delta\alpha^r\right] = 0 \end{cases},$$

where λ_o^g and λ_o^r stand for optimal choices. Then,

$$E\lambda_o^g = \frac{\Delta c^r\Delta b^g + \phi\Delta b^r}{\Delta c^g\Delta c^r - \phi^2} + \frac{\Delta\alpha_1^r\phi + \Delta\alpha^g\Delta c^r}{\Delta c^g\Delta c^r - \phi^2}ED'(e)$$

and

$$E\lambda_o^r = \frac{\phi\Delta b^g + \Delta c^g \Delta b^r}{\Delta c^g \Delta c^r - \phi^2} + \mu \frac{\Delta\alpha^r \Delta c^g + \Delta\alpha^g \phi}{\Delta c^g \Delta c^r - \phi^2} ED'(e). \tag{C.13}$$

C.5 The Representation

This section presents a thorough analysis of benefits and damages under multiple externalities. In parts one to three, we concentrate on benefits. We start by incorporating the sector specific responses (so called cut-off units) into benefit function. This amounts to benefits that depend on the policy instruments: on the price of emissions and on the subsidy thresholds. In addition, we draw a distinction between stochastic and non-stochastic terms in the benefit function. The non-stochastic part is written in Part I, while Part III explains the functional form of the stochastic part. Part II incorporates efficient subsidy rule into non-stochastic part of the benefit function. This leaves us benefits that depends solely on the price of emissions.

In Part IV, we derive a formula for emissions. We follow the steps already familiar from the benefit calculations. This leaves us an emission formula that is divided into stochastic and non-stochastic parts and that depends only on the price of emissions.

C.5.1 Part I

We have

$$B(s, S(s), \theta) = \Psi^1(s, S(s)) + \Psi^2(s, S(s), \theta), \tag{C.14}$$

where Ψ^1 is totally independent of the uncertainties while Ψ^2 is not. In deriving this representation, we find it convenient to write the cut-off units as

$$\lambda_1^g(\theta) = \frac{\Delta b^g + S(s)}{\Delta c^g} - \frac{\Delta\theta^g}{\Delta c^g} + \frac{\Delta\alpha^g}{\Delta c^g} s = \Omega_g - \frac{\Delta\theta^g}{\Delta c^g} + z_g s \tag{C.15}$$

and

$$\lambda_1^r(\theta) = \frac{\Delta b^r - \Delta\theta^r + s\Delta\alpha^r + \phi\lambda_1^{g*}(\theta)}{\Delta c^r} \tag{C.16}$$
$$= \frac{\Delta b^r \Delta c^g + \phi\Delta b^g + \phi S(s)}{\Delta c^r \Delta c^g} - \frac{\Delta c^g \Delta\theta^r + \phi\Delta\theta^g}{\Delta c^r \Delta c^g} + \frac{(\phi\Delta\alpha^g + \Delta c^g \Delta\alpha^r)}{\Delta c^g \Delta c^r} s$$
$$= \Omega_r - \left(\frac{\Delta\theta^r}{\Delta c^r} + \frac{\phi\Delta\theta^g}{\Delta c^r \Delta c^g}\right) + z_r s.$$

We start by deriving the deterministic part of the benefits, Ψ^1. By expanding the benefits, we have (after setting all the uncertainty variables in benefits equal to zero)

$$\Psi^1(s) = z(S(s)) + z_1 s + z_2(S(s))s - \frac{z_3}{2}s^2, \tag{C.17}$$

where

$$z(S(s)) = \Delta b^g \Omega_g - \frac{\Delta c^g}{2}\left(\Omega_g\right)^2 + \Delta b^r \Omega_r - \frac{\Delta c^r}{2}(\Omega_r)^2 + \phi\Omega_g\Omega_r,$$

$$z_1 = \Delta b^g z_g + \Delta b^r z_r,$$

$$z_2(S(s)) = \left(\phi z_r - \Delta c^g z_g\right)\Omega_g + \left(\phi z_g - \Delta c^r z_r\right)\Omega_r,$$

and

$$z_3 = \Delta c^r (z_r)^2 - 2\phi z_g z_r + \Delta c^g \left(z_g\right)^2. \tag{C.18}$$

After inserting $\Omega_g(S(s))$ and $\Omega_r(S(s))$, it holds that

$$z(S(s)) = Z + \frac{1}{\Delta c^r \Delta c^g}\frac{1}{\Delta c^g}\left(\Delta b^r \Delta c^g \phi + \Delta b^g \phi^2\right)S(s)$$
$$- \frac{1}{\Delta c^r \Delta c^g}\frac{1}{\Delta c^g}\left(\left(\Delta c^r \Delta c^g - \phi^2\right)\frac{(S(s))^2}{2}\right)$$
$$= Z + Z' S(s) - Z'' \frac{(S(s))^2}{2},$$

where Z is independent of S and s. It also holds that

$$z_2(S(s)) = -\Delta\alpha^g \frac{\Delta b^g}{\Delta c^g} - \Delta\alpha^r \left(\frac{\Delta b^r \Delta c^g + \Delta b^g \phi}{\Delta c^r \Delta c^g} \right) + \phi z_r \frac{\Delta b^g}{\Delta c^g}$$
$$- \left(\frac{\Delta\alpha^g}{\Delta c^g} + \frac{\Delta\alpha^r \phi}{\Delta c^g \Delta c^r} - \frac{\phi}{\Delta c^g} z_r \right) S(s) = Z_2 - Z_2' S(s).$$

Next, we calculate that

$$\frac{d\Psi^1(s)}{ds} = \frac{dz}{dS}\frac{dS}{ds} + (z_1 + z_2(S(s))) + \frac{dz_2}{dS}\frac{dS}{ds}s - z_3 s.$$

By using the definitions from above, we may also write

$$\frac{d\Psi^1(s)}{ds} = \left(Z' - Z'' S(s)\right)\frac{dS}{ds} + (z_1 + z_2(S(s))) - Z_2'\frac{dS}{ds}s - z_3 s. \tag{C.19}$$

Remember that (by Equations (C.15) and (C.16))

$$z_g = \frac{\Delta\alpha^g}{\Delta c^g} \tag{C.20}$$

and

$$z_r = \frac{\Delta\alpha^r \Delta c^g + \phi\Delta\alpha^g}{\Delta c^r \Delta c^g}, \tag{C.21}$$

so we may (for future needs) calculate that

$$z_1 + Z_2 = \frac{\phi}{\Delta c^g \Delta c^r}\frac{\Delta\alpha^g}{\Delta c^g}(\Delta b^g \phi + \Delta b^r \Delta c^g)$$

and

$$Z_2' = \frac{1}{\Delta c^g \Delta c^r}\frac{\Delta\alpha^g}{\Delta c^g}\left(\Delta c^g \Delta c^r - \phi^2\right).$$

C.5.2 Part II

We will next incorporate efficiency rule into subsidization. We explain in the main text (see Equation (3.28)) that we will apply a linear subsidy rule equal to

$$S(s) = \Gamma + \Gamma_s s = \frac{\phi(\Delta b^r \Delta c^g + \Delta b^g \phi)}{\Delta c^g \Delta c^r - \phi^2} + \frac{\phi(\Delta c^g \Delta\alpha^r + \phi\Delta\alpha^g)}{\Delta c^g \Delta c^r - \phi^2}s. \tag{C.22}$$

This means that

$$\frac{dS}{ds} = \Gamma_s = \frac{\phi(\Delta c^g \Delta \alpha^r + \phi \Delta \alpha^g)}{\Delta c^g \Delta c^r - \phi^2}.$$

Now, we write Equation (C.19) as

$$\frac{d\Psi^1(s)}{ds} = \left(Z'\frac{dS}{ds} + z_1 + Z_2\right) - \left(Z''\frac{dS}{ds} + Z_2'\right)S(s) - \left(Z_2'\frac{dS}{ds} + z_3\right)s$$

or as

$$\frac{d\Psi^1(s)}{ds} = \Psi_0 - \Psi_1 S(s) - \Psi_2 s.$$

Using various definitions and calculations from above, we calculate

$$\begin{aligned}
\Psi_0 &= Z'\frac{dS}{ds} + z_1 + Z_2 \\
&= \frac{1}{\Delta c^r \Delta c^g}\frac{1}{\Delta c^g}\left(\Delta b^r \Delta c^g \phi + \Delta b^g \phi^2\right)\frac{\phi(\Delta c^g \Delta \alpha^r + \phi \Delta \alpha^g)}{\Delta c^g \Delta c^r - \phi^2} \\
&+ \frac{\phi}{\Delta c^g \Delta c^r}\frac{\Delta \alpha^g}{\Delta c^g}(\Delta b^g \phi + \Delta b^r \Delta c^g),
\end{aligned}$$

$$\begin{aligned}
\Psi_1 &= Z''\frac{dS}{ds} + Z_2' \\
&= \frac{\left(\Delta c^r \Delta c^g - \phi^2\right)}{\Delta c^r \Delta c^g}\frac{1}{\Delta c^g}\frac{\phi(\Delta c^g \Delta \alpha^r + \phi \Delta \alpha^g)}{\Delta c^g \Delta c^r - \phi^2} + \frac{1}{\Delta c^g \Delta c^r}\frac{\Delta \alpha^g}{\Delta c^g}\left(\Delta c^g \Delta c^r - \phi^2\right) \\
&= \frac{1}{\Delta c^g}\frac{\phi(\Delta c^g \Delta \alpha^r + \phi \Delta \alpha^g)}{\Delta c^r \Delta c^g} + \frac{1}{\Delta c^g \Delta c^r}\frac{\Delta \alpha^g}{\Delta c^g}\left(\Delta c^g \Delta c^r - \phi^2\right) \\
&= \frac{1}{\Delta c^g}\frac{1}{\Delta c^g \Delta c^r}(\phi \Delta c^g \Delta \alpha^r + \Delta \alpha^g (\Delta c^g \Delta c^r)) = \frac{\phi \Delta \alpha^r + \Delta \alpha^g \Delta c^r}{\Delta c^g \Delta c^r},
\end{aligned}$$

and

$$\Psi_2 = Z_2'\frac{dS}{ds} + z_3 = \frac{1}{\Delta c^g \Delta c^r}\frac{\Delta \alpha^g}{\Delta c^g}\phi(\Delta c^g \Delta \alpha^r + \phi \Delta \alpha^g) + z_3.$$

After simplifying, we can write

$$\Psi_0 = \frac{\Delta b^r \Delta c^g \phi + \Delta b^g \phi^2}{\Delta c^r \Delta c^g}\frac{\phi \Delta \alpha^r + \Delta \alpha^g \Delta c^r}{\Delta c^g \Delta c^r - \phi^2}.$$

As for the factor Ψ_2, we insert z_g and z_r (Equations (C.20) and (C.21)) into the factor z_3 (Equation (C.18)), so that

$$\begin{aligned} z_3 &= \Delta c^r (z_r)^2 - 2\phi z_g z_r + \Delta c^g \left(z_g\right)^2 \\ &= \frac{1}{\Delta c^g \Delta c^r}\left(\Delta c^g (\Delta\alpha^r)^2 + \Delta c^r (\Delta\alpha^g)^2 - \frac{\phi^2}{\Delta c^g}(\Delta\alpha^g)^2\right). \end{aligned}$$

Thus,

$$\begin{aligned} \Psi_2 &= \frac{1}{\Delta c^g \Delta c^r}\frac{\Delta\alpha^g}{\Delta c^g}\phi(\Delta c^g \Delta\alpha^r + \phi\Delta\alpha^g) \\ &\quad + \frac{1}{\Delta c^g \Delta c^r}\left(\Delta c^g (\Delta\alpha^r)^2 + \Delta c^r (\Delta\alpha^g)^2 - \frac{\phi^2}{\Delta c^g}(\Delta\alpha^g)^2\right) \\ &= \frac{1}{\Delta c^g \Delta c^r}\left(\phi\Delta\alpha^g\Delta\alpha^r + \Delta c^g (\Delta\alpha^r)^2 + \Delta c^r (\Delta\alpha^g)^2\right). \end{aligned}$$

Next, we insert the rule $S(s)$ (Equation (C.22)) into $\frac{d\Psi^1(s)}{ds}$. It now holds that

$$\begin{aligned} &\Psi_0 - \Psi_1 S(s) \\ &= \frac{\Delta b^r \Delta c^g \phi + \Delta b^g \phi^2}{\Delta c^r \Delta c^g}\frac{\phi\Delta\alpha^r + \Delta\alpha^g \Delta c^r}{\Delta c^g \Delta c^r - \phi^2} \\ &- \frac{\phi\Delta\alpha^r + \Delta\alpha^g \Delta c^r}{\Delta c^g \Delta c^r}\left(\frac{\phi(\Delta b^r \Delta c^g + \Delta b^g \phi)}{\Delta c^g \Delta c^r - \phi^2} + \frac{\phi(\Delta c^g \Delta\alpha^r + \phi\Delta\alpha^g)}{\Delta c^g \Delta c^r - \phi^2}s\right) \\ &= -\frac{\phi\Delta\alpha^r + \Delta\alpha^g \Delta c^r}{\Delta c^r \Delta c^g}\left(\frac{\phi(\Delta c^g \Delta\alpha^r + \phi\Delta\alpha^g)}{\Delta c^g \Delta c^r - \phi^2}s\right). \end{aligned}$$

Furthermore, if we write

$$\Psi_2 \equiv \frac{\Psi_2 \Delta c^g \Delta c^r - \Psi_2 \phi^2}{\Delta c^g \Delta c^r - \phi^2},$$

then

$$\Psi_0 - \Psi_1 S(s) - \Psi_2 s = -\frac{(\phi\Delta\alpha^r + \Delta\alpha^g \Delta c^r)\left(\phi\Delta c^g \Delta\alpha^r + \phi^2 \Delta\alpha^g\right)}{(\Delta c^r \Delta c^g)(\Delta c^g \Delta c^r - \phi^2)} s$$
$$+ \frac{\phi^2\left(\phi\Delta\alpha^g \Delta\alpha^r + \Delta c^g (\Delta\alpha^r)^2 + \Delta c^r (\Delta\alpha^g)^2\right)}{(\Delta c^r \Delta c^g)(\Delta c^g \Delta c^r - \phi^2)} s$$
$$- \frac{\Delta c^g \Delta c^r \left(\phi\Delta\alpha^g \Delta\alpha^r + \Delta c^g (\Delta\alpha^r)^2 + \Delta c^r (\Delta\alpha^g)^2\right)}{(\Delta c^r \Delta c^g)(\Delta c^g \Delta c^r - \phi^2)} s.$$

After simplifying, we have

$$\Psi_0 - \Psi_1 S(s) - \Psi_2 s = -\left(\frac{\Delta c^g (\Delta\alpha^r)^2 + 2\phi\Delta\alpha^g \Delta\alpha^r + \Delta c^r (\Delta\alpha^g)^2}{\Delta c^g \Delta c^r - \phi^2}\right) s = -\frac{s}{c}, \tag{C.23}$$

where the parameter c is defined in the main text (Equation (3.20)). We may then write

$$\Psi^1(s) = \overline{\Psi}^1 - \frac{1}{2c} s^2. \tag{C.24}$$

C.5.3 Part III

In this part, we derive a representation for $\Psi^2(s,\theta)$ in Equation (C.14). To begin with, we collect all the benefit-related factors that are not included in $\Psi^1(s)$. We write these as

$$\Psi^2(\theta) = \Delta b^g \left(-\frac{\Delta\theta^g}{\Delta c^g}\right) + \Delta b^r \left(-\frac{1}{\Delta c^r}\Delta\theta^r - \frac{\phi}{\Delta c^r \Delta c^g}\Delta\theta^g\right)$$
$$- \frac{\Delta c^g}{2}\left(\frac{\Delta\theta^g}{\Delta c^g}\right)^2 - \frac{\Delta c^r}{2}\left(\frac{1}{\Delta c^r}\Delta\theta^r + \frac{\phi}{\Delta c^r \Delta c^g}\Delta\theta^g\right)^2$$
$$+ \phi\left(\frac{\Delta\theta^g}{\Delta c^g}\left(\frac{1}{\Delta c^r}\Delta\theta^r + \frac{\phi}{\Delta c^r \Delta c^g}\Delta\theta^g\right)\right)$$
$$+ \frac{(\Delta\theta^g)^2}{\Delta c^g} + \Delta\theta^r\left(\frac{1}{\Delta c^r}\Delta\theta^r + \frac{\phi}{\Delta c^r \Delta c^g}\Delta\theta^g\right),$$

so $\Psi^2(\theta)$ include factors in $\Psi^2(s,\theta)$ that do not contain the price s. Then,

$$
\begin{aligned}
\Psi^2(s,\theta) &= \Psi^2(\theta)+\left(\Delta c^g\frac{\Delta\theta^g}{\Delta c^g}-\phi\left(\frac{1}{\Delta c^r}\Delta\theta^r+\frac{\phi}{\Delta c^r\Delta c^g}\Delta\theta^g\right)-\Delta\theta^g\right)\Omega_g\\
&+\left(\Delta c^r\left(\frac{1}{\Delta c^r}\Delta\theta^r+\frac{\phi}{\Delta c^r\Delta c^g}\Delta\theta^g\right)-\phi\left(\frac{\Delta\theta^g}{\Delta c^g}\right)-\Delta\theta^r\right)\Omega_r\\
&+\left(\Delta c^g\frac{\Delta\theta^g}{\Delta c^g}z_g s+\left(\Delta\theta^r+\frac{\phi}{\Delta c^g}\Delta\theta^g\right)z_r\right)s\\
&-\left(\phi\left(z_r\frac{\Delta\theta^g}{\Delta c^g}+z_g\left(\frac{1}{\Delta c^r}\Delta\theta^r+\frac{\phi}{\Delta c^r\Delta c^g}\Delta\theta^g\right)\right)-\Delta\theta^g z_g-\Delta\theta^r z_r\right)s.
\end{aligned}
$$

Actually, many factors will cancel out, so we have

$$
\begin{aligned}
\Psi^2(s,\theta) &= \Psi^2(\theta)-\phi\left(\frac{1}{\Delta c^r}\Delta\theta^r+\frac{\phi}{\Delta c^r\Delta c^g}\Delta\theta^g\right)\left(\frac{\Delta b^g}{\Delta c^g}+\frac{S}{\Delta c^g}\right)\\
&-\phi\left(z_g s\left(\frac{1}{\Delta c^r}\Delta\theta^r+\frac{\phi}{\Delta c^r\Delta c^g}\Delta\theta^g\right)\right)s.
\end{aligned}
$$

Denote

$$
y(\theta)=\Psi^2(\theta)-\phi\left(\frac{1}{\Delta c^r}\Delta\theta^r+\frac{\phi}{\Delta c^r\Delta c^g}\Delta\theta^g\right)\frac{\Delta b^g}{\Delta c^g},
$$

so

$$
\begin{aligned}
\Psi^2(s,\theta) = y(\theta)&-\phi\left(\frac{1}{\Delta c^r}\Delta\theta^r+\frac{\phi}{\Delta c^r\Delta c^g}\Delta\theta^g\right)\left(\frac{S}{\Delta c^g}\right)\\
&-\phi\left(z_g\left(\frac{1}{\Delta c^r}\Delta\theta^r+\frac{\phi}{\Delta c^r\Delta c^g}\Delta\theta^g\right)\right)s.
\end{aligned}
$$

Remember that

$$
z_g=\frac{\Delta\alpha^g}{\Delta c^g},
$$

so

$$
\Psi^2(s,\theta)=y(\theta)-\frac{\phi}{\Delta c^g\Delta c^r}\left(\Delta\theta^r+\frac{\phi}{\Delta c^g}\Delta\theta^g\right)(S+\Delta\alpha^g s).
$$

If we further let

$$
y_1(\theta)=\frac{\phi}{\Delta c^g\Delta c^r}\left(\Delta\theta^r+\frac{\phi}{\Delta c^g}\Delta\theta^g\right), \tag{C.25}
$$

then

$$\Psi^2(s,\theta) = y(\theta) - y_1(\theta)(S(s) + \Delta\alpha^g s). \tag{C.26}$$

C.5.4 Part IV

Before concluding, we derive the emissions under efficient subsidization. The level of emissions is (by Equation 3.16)

$$e = \lambda_0^{g+}\alpha_0^g - \Delta\alpha^g \lambda_1^g + \lambda_0^{r+}\alpha_0^r - \Delta\alpha^r \lambda_1^r.$$

We incorporate the units λ_1^g and λ_1^r from Equations (3.30) and (3.31) into the formula, so

$$\begin{aligned} e(s,\theta) = {} & x + \left(\frac{\Delta\alpha^g \Delta c^r + \Delta\alpha^r \phi}{\Delta c^r \Delta c^g}\right)\Delta\theta^g + \frac{\Delta\alpha^r}{\Delta c^r}\Delta\theta^r \\ & - \left(\frac{\Delta\alpha^g \Delta c^r + \Delta\alpha^r \phi}{\Delta c^r \Delta c^g}\right)S(s) - \left(z_r \Delta\alpha^r + \Delta\alpha^g z_g\right)s \\ = {} & x + x(\theta) - x_1 S(s) - x_2 s. \end{aligned} \tag{C.27}$$

The factor x includes terms that does not contain price or uncertainty variables.

Next, we incorporate the rule $S(s)$ (Equation (C.22)) into emissions. In particular, we have

$$\begin{aligned} x_1 S(s) &= x_1\left(\Gamma + \frac{\phi(\Delta c^g \Delta\alpha^r + \phi\Delta\alpha^g)}{\Delta c^g \Delta c^r - \phi^2}s\right) = \\ &= x_1\Gamma + \left(\frac{\Delta\alpha^g \Delta c^r + \Delta\alpha^r \phi}{\Delta c^r \Delta c^g}\right)\frac{\phi(\Delta c^g \Delta\alpha^r + \phi\Delta\alpha^g)}{\Delta c^g \Delta c^r - \phi^2}s. \end{aligned}$$

Furthermore, as

$$\begin{aligned} x_2 &= z_r \Delta\alpha^r + \Delta\alpha^g z_g = \left(\frac{\Delta c^g \Delta\alpha^r + \phi\Delta\alpha^g}{\Delta c^g \Delta c^r}\right)\Delta\alpha^r + \Delta\alpha^g \frac{\Delta\alpha^g}{\Delta c^g} \\ &= \frac{\Delta c^g (\Delta\alpha^r)^2 + \phi\Delta\alpha^r \Delta\alpha^g + \Delta c^r (\Delta\alpha^g)^2}{\Delta c^g \Delta c^r}, \end{aligned}$$

we have

$$e(s,\theta)=X+x(\theta)-\left(\frac{\Delta\alpha^g\Delta c^r+\Delta\alpha^r\phi}{\Delta c^r\Delta c^g}\right)\frac{\phi(\Delta c^g\Delta\alpha^r+\phi\Delta\alpha^g)}{\Delta c^g\Delta c^r-\phi^2}s$$
$$-\frac{\Delta c^g(\Delta\alpha^r)^2+\phi\Delta\alpha^r\Delta\alpha^g+\Delta c^r(\Delta\alpha^g)^2}{\Delta c^g\Delta c^r}s,$$

where

$$X=x-x_1\Gamma$$

is a constant. We write that

$$x_2=\frac{\Delta c^g\Delta c^r x_2-\phi^2x_2}{\Delta c^g\Delta c^r-\phi^2},$$

so

$$e(s,\theta)=X+x(\theta)-\frac{\Delta\alpha^g\Delta c^r+\Delta\alpha^r\phi}{\Delta c^r\Delta c^g}\frac{\phi(\Delta c^g\Delta\alpha^r+\phi\Delta\alpha^g)}{\Delta c^g\Delta c^r-\phi^2}s$$
$$-\frac{\Delta c^g\Delta c^r\left(\Delta c^g(\Delta\alpha^r)^2+\phi\Delta\alpha^r\Delta\alpha^g+\Delta c^r(\Delta\alpha^g)^2\right)}{\Delta c^g\Delta c^r(\Delta c^g\Delta c^r-\phi^2)}s$$
$$+\frac{\phi^2\left(\Delta c^g(\Delta\alpha^r)^2+\phi\Delta\alpha^r\Delta\alpha^g+\Delta c^r(\Delta\alpha^g)^2\right)}{\Delta c^g\Delta c^r(\Delta c^g\Delta c^r-\phi^2)}s.$$

Many factors will cancel out, so we finally have

$$\begin{aligned}e(s,\theta)&=X+x(\theta)+\frac{\Delta c^g(\Delta\alpha^r)^2+2\phi\Delta\alpha^r\Delta\alpha^g+\Delta c^r(\Delta\alpha^g)^2}{\Delta c^g\Delta c^r-\phi^2}s \qquad \text{(C.28)}\\&=X+x(\theta)+\frac{s}{c},\end{aligned}$$

where the factor c is defined in Equation (3.20) in the main text.

C.6 Proof of Lemma 3

We claim in Lemma 3 (Section 3.4.6) that $r_1=\frac{w^2}{cc_{sb}}>1$. As for the proof, note first that

$$w = \frac{\Delta c^g \Delta c^r}{\Delta c^g (\Delta \alpha^r)^2 + \phi \Delta \alpha^r \Delta \alpha^g + \Delta c^r (\Delta \alpha^g)^2}, \tag{C.29}$$

$$c = \frac{\Delta c^g \Delta c^r - \phi^2}{\Delta c^r (\Delta \alpha^g)^2 + 2\phi \Delta \alpha^g \Delta \alpha^r + \Delta c^g (\Delta \alpha^r)^2}, \tag{C.30}$$

and

$$c_{sb} = \frac{\Delta c^g \Delta c^r}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 - \frac{\phi^2}{\Delta c^g} (\Delta \alpha^g)^2}.$$

It holds that

$$\frac{1}{c}\frac{1}{c_{sb}} = \frac{\Delta c^r (\Delta \alpha^g)^2 + \Delta c^g (\Delta \alpha^r)^2 + 2\phi \Delta \alpha^g \Delta \alpha^r}{(\Delta c^g \Delta c^r)^2}$$
$$\frac{\Delta c^g (\Delta \alpha^r)^2 + \left(1 - \frac{\phi^2}{\Delta c^g \Delta c^r}\right) \Delta c^r (\Delta \alpha^g)^2}{1 - \frac{\phi^2}{\Delta c^g \Delta c^r}}.$$

Thus,

$$r_1 = \frac{w^2}{c c_{sb}} = \frac{\Delta c^r (\Delta \alpha^g)^2 + \Delta c^g (\Delta \alpha^r)^2 + 2\phi \Delta \alpha^g \Delta \alpha^r}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 + \phi \Delta \alpha^r \Delta \alpha^g}$$
$$\frac{\Delta c^g (\Delta \alpha^r)^2 + \left(1 - \frac{\phi^2}{\Delta c^g \Delta c^r}\right) \Delta c^r (\Delta \alpha^g)^2}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 + \phi \Delta \alpha^r \Delta \alpha^g} \frac{1}{\left(1 - \frac{\phi^2}{\Delta c^g \Delta c^r}\right)}.$$

After some straightforward manipulations, we may write that

$$r_1 = \left(1 + \frac{\phi \Delta \alpha^g \Delta \alpha^r}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 + \phi \Delta \alpha^r \Delta \alpha^g}\right)$$
$$\left(1 - \frac{\phi \Delta \alpha^r \Delta \alpha^g - (h-1) \Delta c^g (\Delta \alpha^r)^2}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 + \phi \Delta \alpha^r \Delta \alpha^g}\right),$$

where

$$h = \frac{1}{1 - \frac{\phi^2}{\Delta c^g \Delta c^r}} = \frac{\Delta c^g \Delta c^r}{\Delta c^g \Delta c^r - \phi^2} > 1.$$

We further define

$$h_1 = \frac{\phi \Delta \alpha^g \Delta \alpha^r}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 + \phi \Delta \alpha^r \Delta \alpha^g} > 0$$

and

$$h_2 = \frac{(h-1) \Delta c^g (\Delta \alpha^r)^2}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 + \phi \Delta \alpha^r \Delta \alpha^g} > 0,$$

so that

$$r_1 = (1 + h_1)(1 - h_1 + h_2).$$

Specifically, as long as

$$(1 + h_1)(1 - h_1 + h_2) > 1,$$

then $r_1 > 1$. Consequently, as long as

$$h_2 > \frac{(h_1)^2}{1 + h_1},$$

then $r_1 > 1$. We calculate that

$$h_2 - \frac{(h_1)^2}{1 + h_1} = \frac{\phi^2}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 + \phi \Delta \alpha^r \Delta \alpha^g}$$
$$\left(\frac{\Delta c^g (\Delta \alpha^r)^2}{\Delta c^g \Delta c^r - \phi^2} - \frac{(\Delta \alpha^g \Delta \alpha^r)^2}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2} \right),$$

or, after some manipulations, we may write that

$$h_2 - \frac{(h_1)^2}{1 + h_1} = \frac{\phi^2 (\Delta \alpha^r)^2}{\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2 + \phi \Delta \alpha^r \Delta \alpha^g}$$
$$\frac{(\Delta c^g \Delta \alpha^r)^2 + \phi^2 (\Delta \alpha^g)^2}{(\Delta c^g \Delta c^r - \phi^2)\left(\Delta c^g (\Delta \alpha^r)^2 + \Delta c^r (\Delta \alpha^g)^2\right)},$$

which is clearly strictly larger than zero. We conclude that $r_1 > 1$.